Test 1

ⓘ This is a preview of the published version of the quiz

Started: Feb 20 at 5:56pm

Quiz Instructions

Choose the best answer for each problem.

Question 1

2 pts

Suppose $\lim_{x \to c} f(x) = L$. Which one of the following must be true?

○ $f(x)$ is defined at $x = c$.

○ $f(c) = L$

○ Substituting c for x results in the form $0/0$.

✱ $f(x)$ is defined on both sides of $x = c$. ← DEFINITION SAYS "OPEN INTERVAL CONTAINING THE NUMBER C, BUT f NEED NOT BE DEFINED AT C."

Question 2

2 pts

Suppose you were asked to use a table of values to estimate the limit of $f(x)$ at $x = 3$. Which list of x-values shown below would be best for your table?

○ $x = 3.01, 3.001, 3.0001, 3, 2.99, 2.999, 2.9999$ $\xleftarrow{\text{No}}$ X=3 SHOULD BE IN TABLE

○ $x = 0, 1, 2, 3, 4, 5$ $\xleftarrow{\text{No}}$ NOT CLOSE TO X=3

✱ $x = 3.1, 3.01, 3.001, 2.9, 2.99, 2.999$ ← LOOKS GOOD !

○ $x = 3.1, 3.01, 3.001, 3.0001, 2.9$ $\xleftarrow{\text{No}}$ NOT ENOUGH VALUES TO LEFT OF X=3

Question 3

2 pts

Which one of these is true about the limit concept?

- ○ If a function is defined, then it must have a limit. Nope!
- ○ The limit tells about about the value of the function at the limit point. Nope!
- ✱ The limit tells us about the behavior of the function near the limit point.
- ○ To obtain the limit, you substitute the limit point into the function.

NOT NECESSARILY

Question 4

5 pts

Use a table of numerical values to estimate the limit: $\lim_{x \to 0} \frac{5^{2x} - 1}{6x}$

- ○ 0.8333
- ✱ 0.5365
- ○ The limit does not exist.
- ○ 0.5000

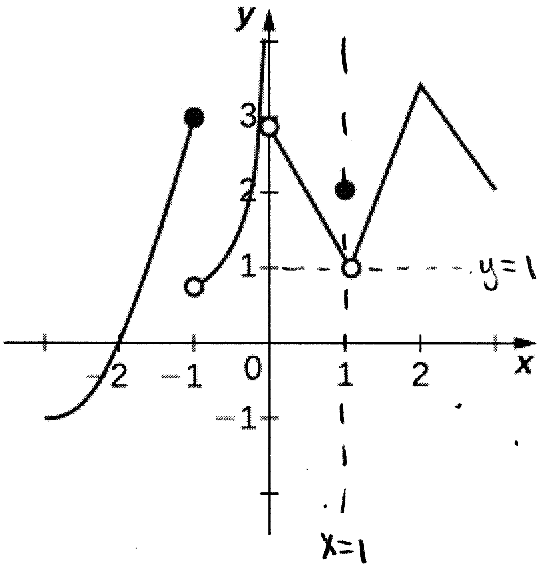
x	f(x)
0.1	0.6329
-0.1	0.4587
0.01	0.5452
-0.01	0.5279
0.001	0.5373
-0.001	0.5356
0.0001	0.5366
-0.0001	0.5364

0.5365 LOOKS LIKE THE BEST ANSWER.

Question 5

3 pts

The graph of the function f is shown below. Use the graph to estimate $\lim_{x \to 1} f(x)$.



$\lim_{x \to 1} f(x) = 1$

○ 2

○ 3

○ The limit does not exist.

○ 1

Question 6

2 pts

Suppose $\lim_{t \to 4} f(t) = 7$. Which statement below is correct?

○ $f(4)$ cannot be equal to 7. $\xleftarrow{\text{No}}$ IT MIGHT BE, e.g., IF f IS CONT.

○ $f(4)$ must be equal to 7. $\xleftarrow{\text{No}}$ NOT NECESSARILY

○ $f(4)$ cannot be defined. $\xleftarrow{\text{No}}$ IT MIGHT BE.

○ $f(4)$ may or may not be defined.

Question 7

3 pts

Which one of these limits fails to exist because the limit from the left does not equal the limit from the right?

○ $\lim_{x \to 5} \frac{|x-5|}{(x-5)^2}$ $\qquad \lim_{x \to 5} \frac{|x-5|}{(x-5)^2} = +\infty$

✱ $\lim_{x \to 0} \frac{\sqrt{x^2}}{x^2+x}$ $\qquad \lim_{x \to 0^-} \frac{\sqrt{x^2}}{x^2+x} = -1, \quad \lim_{x \to 0^+} \frac{\sqrt{x^2}}{x^2+x} = 1$

○ $\lim_{x \to 2} f(x)$, where $f(x) = \begin{cases} 5x+2, & x < 2 \\ x^2+x+6, & x > 2 \end{cases}$ $\qquad \lim_{x \to 2} f(x) = 12$

○ $\lim_{x \to 0} \sqrt{x+5}$ $\qquad \lim_{x \to 0} \sqrt{x+5} = \sqrt{5}$

Question 8

3 pts

Explain why this limit fails to exist: $\lim_{x \to 0} \cos\left(\frac{1}{x^2}\right)$

○ The limit from the left does not equal the limit from the right.

○ The function values grow without bound as the limit point is approached.

✱ The function values oscillate as the limit point is approached.

○ The function is not defined on both sides of the limit point.

THIS IS A CLASSIC EXAMPLE OF THIS. AS $x \to 0$, THE COSINE OSCILLATES THROUGH MORE AND MORE PERIODS.

Question 9

3 pts

Explain why this limit fails to exist: $\lim_{x \to 0} \frac{x}{\ln x}$

✱ The function is not defined on both sides of the limit point.

$\ln x$ IS DEFINED ONLY FOR $x > 0$.

○ The function values grow without bound as the limit point is approached.

○ The function values oscillate as the limit point is approached.

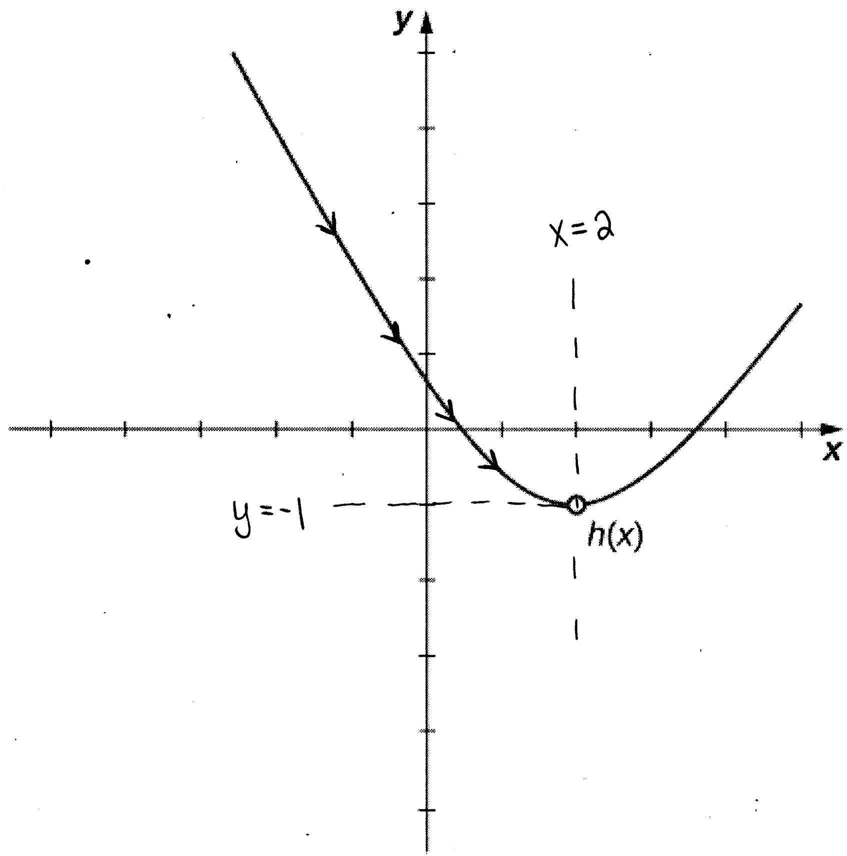
○ The limit from the left does not equal the limit from the right.

Question 10

3 pts

The graph of the function h is shown below. Use the graph to estimate $\lim_{x \to 2^+} h(x)$.

Assume each tick mark on the graph represents one unit.



○ 0.75

○ 0

○ The limit does not exist.

○ −1

Question 11

2 pts

Suppose $\lim_{x \to 6} f(x) = -9$. Which one of the following must be true?

AUTOMATICALLY TELLS US

$$\lim_{x \to 6^-} f(x) = \lim_{x \to 6^+} f(x) = -9$$

- None of these
- $\lim_{x \to 6^+} f(x) = 9$
- $\lim_{x \to -6} f(x) = 9$
- $\lim_{x \to 6^-} f(x) = -9$

Question 12

5 pts

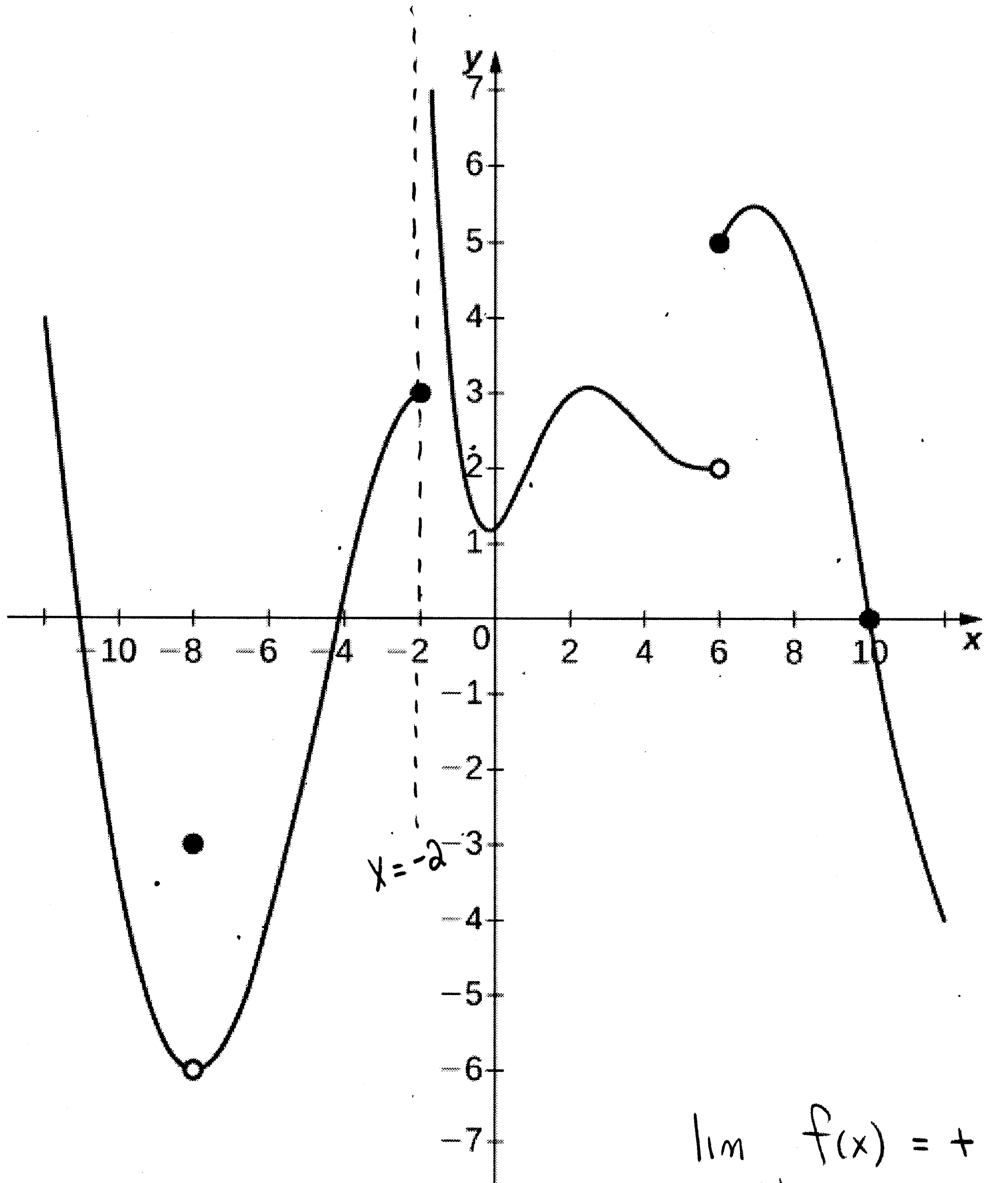
Evaluate the limit: $\lim_{w \to 7^-} \dfrac{w^2 - 49}{w + 7} = \dfrac{0}{14}$ BY DIRECT SUBS.

- 0
- -14
- The limit does not exist.
- $0/0$

Question 13

3 pts

The graph of the function f is shown below. Use the graph to estimate $\lim_{x \to -2^+} f(x)$.



$$\lim_{x \to -2^+} f(x) = +\infty$$

- ○ 5
- ○ 6
- ✱ $+\infty$
- ○ 3

Question 14 **6 pts**

0/0 more work!

Evaluate the limit: $\displaystyle \lim_{y \to 2} \frac{5y - 10}{\sqrt{y} - \sqrt{2}} \cdot \frac{\sqrt{y}+\sqrt{2}}{\sqrt{y}+\sqrt{2}} = \lim_{y\to 2} \frac{5\cancel{(y-2)}(\sqrt{y}+\sqrt{2})}{\cancel{y-2}}$

$= 5(2\sqrt{2})$

- $-\sqrt{2}$
- $10\sqrt{2}$
- $0/0$
- $6\sqrt{2}$

Question 15

6 pts

0/0 More work.

Evaluate the limit: $\lim_{x \to 1} \dfrac{\sin(2x-2)}{\pi x - \pi}$ $= \lim_{x \to 1} \dfrac{2}{\pi} \dfrac{\sin(2x-2)}{2x-2} = \dfrac{2}{\pi}$

- 0
- None of these
- $2/\pi$
- 1

Question 16

6 pts

0/0 More work.

Evaluate the limit: $\lim_{x \to -3^+} \left(\dfrac{x^2 - x - 12}{x^2 + 8x + 15} \right)$

$\lim_{x \to -3^+} \dfrac{\cancel{(x+3)}(x-4)}{\cancel{(x+3)}(x+5)} = \dfrac{-7}{2}$

- 0
- $-7/8$
- $-7/2$
- The limit does not exist.

Question 17

6 pts

Determine the value of c so that f is continuous everywhere.

$$f(x) = \begin{cases} x^2 + cx + 7, & x \le 3 \\ 3x^2 - 1, & x > 3 \end{cases}$$

TO MAKE $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$,

WE NEED

$9 + 3c + 7 = 3(9) - 1$

$3c + 16 = 26$

$3c = 10$

$c = 10/3$

- ○ There is no value of c that will make the function continuous.
- ○ $c = 64/3$
- ✱ $c = 10/3$
- ○ c can be any real number.

Question 18

4 pts

Let $f(x) = \dfrac{3\tan x}{6x}$. Which of these is a vertical asymptote of the graph of f?

- ○ $x = \pi$ ← NO $\lim_{x \to \pi} \frac{3 \tan x}{6x} = \frac{0}{6\pi} = 0$
- ✱ $x = \pi/2$ ← YES GRAPH OF TANGENT FUNCTION HAS V.A. AT $x = \frac{\pi}{2}$.
- ○ None of these
- ○ $x = 0$ ← NO $\lim_{x \to 0} \frac{3 \tan x}{6x} = \frac{3}{6} = \frac{1}{2}$

Question 19

5 pts

The graph of f does NOT have a vertical asymptote at $x = 5$. Find the value of c.

$$f(x) = \frac{2x^2 + cx + 20}{x - 5}$$

$x = 5$ MUST BE A REMOVABLE DISCONT.

$2x^2 + cx + 20 = 0$ WHEN $x = 5$

$50 + 5c + 20 = 0 \Rightarrow c = -14$

- ○ $c = 0$

$c = -14$

$c = -7$

$c = 14$

Question 20

3 pts

Which one of these functions is NOT continuous at $x = 2$?

$f(x) = \dfrac{x^2 - 2}{x^2 - x - 2}$ $\quad x^2 - x - 2 = (x-2)(x+1)$

$f(x) = \dfrac{x^2 + \pi \cos x}{\sqrt{xe^x}}$

$f(x) = \tan x$

$f(x) = 13$

Question 21

4 pts

Find and classify the discontinuity of $g(x) = \dfrac{x^2 - 9}{x + 3}$. $= \dfrac{(x+3)(x-3)}{(x+3)}$

There are no discontinuities.

X = -3 IS A REMOVABLE DISCONT.

$x = -3$ (Infinite)

$x = -3$ (Removable)

$x = 3$ (Infinite)

Question 22

4 pts

Suppose that $\lim_{x \to 1} \dfrac{x^2 - 1}{g(x)} = 37$. Assuming it exists, determine the value of $\lim_{x \to 1} g(x)$.

SINCE THE NUMERATOR APPROACHES ZERO, THE DENOMINATOR MUST APPROACH ZERO (0/0 FORM). OTHERWISE THE LIMIT WOULD BE ZERO.

- ○ $\lim_{x \to 1} g(x) = -1$
- ● $\lim_{x \to 1} g(x) = 0$
- ○ $\lim_{x \to 1} g(x) = 1$
- ○ $\lim_{x \to 1} g(x) = 37$

Question 23 — 2 pts

True or false: If $p(x)$ is a polynomial function, then the limit at any point can ALWAYS be determined by direct substitution.

- ● True
- ○ False

Question 24 — 2 pts

True or false: If f is continuous at $x = -2$, then $\lim_{x \to -2^-} f(x) = f(-2)$.

- ● True
- ○ False

IN FACT, $\lim_{x \to -2} f(x) = f(-2)$.

Question 25

3 pts

Suppose that $f(5) = 9$ and that f is NOT continuous at $x = 5$. Which one of these CANNOT be true?

- ○ $\lim_{x \to 5} f(x)$ does not exist
- ✱ $\lim_{x \to 5} f(x) = 9$ ← IF THIS WOULD BE TRUE, f WOULD BE CONTINUOUS AT X = 5
- ○ $\lim_{x \to 5^-} f(x) = 9$
- ○ $\lim_{x \to 5^+} f(x) = 9$

Question 26

4 pts

Suppose that $\lim_{y \to 7} h(y)$ exists and that $\lim_{y \to 7} \left(\dfrac{2y - h(y)}{\dfrac{1}{7} - \dfrac{1}{y}} \right)$ also exists. Determine $\lim_{y \to 7} h(y)$.

SINCE THE DENOM. APPROACHES ZERO, THE ONLY WAY THIS LIMIT COULD POSSIBLY EXIST IS IF THE NUMER. ALSO APPROACHES ZERO.

- ○ $\lim_{y \to 7} h(y) = 7$
- ○ $\lim_{y \to 7} h(y) = 0$
- ○ None of these
- ✱ $\lim_{y \to 7} h(y) = 14$

Question 27

2 pts

True or false: The limit of any trigonometric function can ALWAYS be determined by direct substitution.

$\lim_{X \to \frac{\pi}{2}} \text{TAN } X \neq \text{TAN } \frac{\pi}{2}$

- ○ True
- ✱ False

Question 28

5 pts

The function f has two discontinuities. Find and classify them.

$$f(x) = \begin{cases} (x^2-4)/(x-2), & x < 2 \\ 3x-4, & 2 \leq x < 5 \\ x^2-14, & x > 5 \end{cases}$$

- ○ $x = 2$ (Infinite) and $x = 5$ (Jump)
- ○ $x = 2$ (Removable) and $x = 5$ (Removable)
- ✱ $x = 2$ (Jump) and $x = 5$ (Removable)
- ○ $x = 2$ (Jump) and $x = 5$ (Jump)

$\underline{X = 2}$

$\lim_{X \to 2^-} f(x) = \lim_{X \to 2^-} \frac{(X-2)(X+2)}{X-2}$

$= 4$

$\lim_{X \to 2^+} f(x) = \lim_{X \to 2^+} 3X-4 = 2$

JUMP DISCONT AT $X = 2$

$\underline{X = 5}$

$f(5)$ IS NOT DEFINED,

BUT $\lim_{X \to 5^-} f(x) = \lim_{X \to 5^-} 3X-4$
$= 11$

Not saved Submit Quiz

AND $\lim_{X \to 5^+} f(x) = \lim_{X \to 5^+} X^2 - 14$

$= 11$

REMOVABLE DISCONT AT $X = 5$