Math 131 - Test 3

April 14, 2022

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) The function $f(x) = 2x^3 + x - 3$ has an inverse function. Call it g. Find $g'(0)$ and $g'(-3)$.

2. (6 points) Compute the slope of the line tangent to the graph of $y = x \sin^{-1}(2x)$ at the point where $x = 1/4$. Write your answer in exact form, simplified as much as possible.

3. (6 points) Let $g(x) = e^{-x^2}$. Find $g''(x)$.

4. (6 points) Let $f(x) = \log_3[(8x^2 + x)^4]$. Compute $f'(1)$. Write your final answer in decimal form rounded to the nearest thousandth.

5. (8 points) Use logarithmic differentiation to find $\dfrac{dy}{dx}$ when $y = \dfrac{x\cos x}{e^{5x}(x^2+1)^3}$.

6. (6 points) Some values of $f(x)$ and $f'(x)$ near $x = 2$ are given in the table below.

x	1.50	1.75	2.00	2.25	2.50
$f(x)$	6.08	6.90	8.00	9.41	11.14
$f'(x)$	2.74	3.82	5.00	6.26	7.60

Determine the linearization of f at $x = 1.75$, and use it to approximate $f(1.81)$.

7. (10 points) Determine the differential dy.

   (a) $y = 5^{x^2+1}$

   (b) $y = \cot^{-1}(\sqrt{x})$

8. (6 points) Use differentials to approximate the change in $y = \dfrac{1}{1-x}$ as x changes from 2 to 1.98.

9. (6 points) Suppose that the percent error in measuring the side length of a cube is 2%. Use differentials to estimate the percent error in computing the cube's volume.

10. (6 points) Find the critical numbers of $g(x) = \dfrac{x^2}{x-1}$.

11. (8 points) Use calculus techniques to find the absolute minimum and maximum values of $f(x) = x^4 - 2x^2 + 1$ on $[0, 2]$.

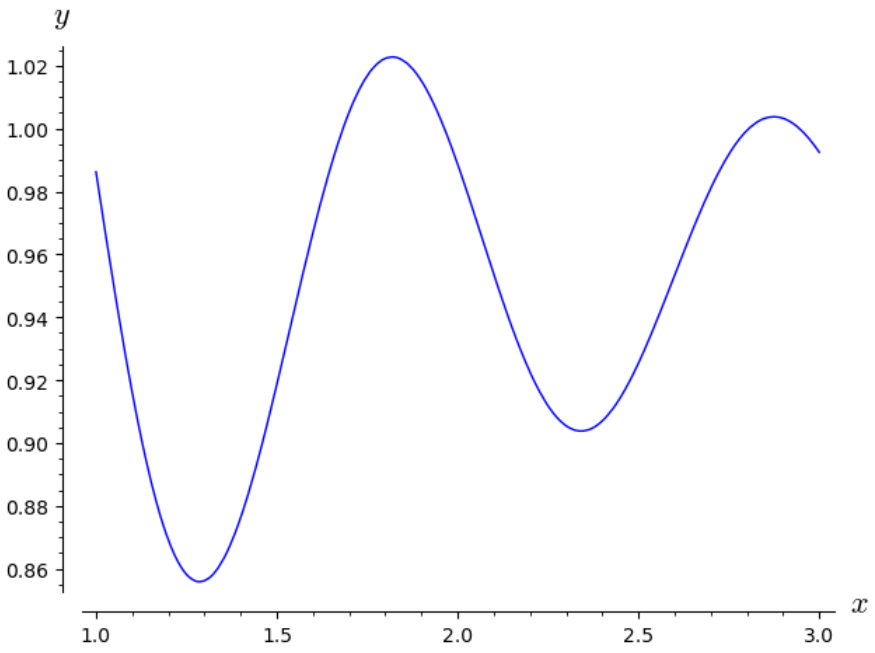
12. (8 points) Use calculus techniques to find the absolute minimum and maximum values of $g(x) = \frac{1}{2}x - x^{2/3}$ on $[-1, 4]$.

13. (10 points) Use calculus techniques to find open intervals on which

$$f(x) = 2x^3 - 9x^2 + 12x - 5$$

is increasing/decreasing. Also identify all relative extreme values.

14. (2 points) The graph of the function f is shown below. Sketch the graph of the linearization of f at $x = 2.5$.



15. (6 points) The functions $f(x)$ and $f'(x)$ are defined for all x. Furthermore, $f'(x)$ has exactly three zeros: $x = -3$, $x = 5$, and $x = 7$. Use this and the information below to find the locations (x-values) of all relative extrema.

x	-12	-6	0	6	12
$f'(x)$	3	8	2	1	-5