Math 131 - Final Exam

Name ______________________________

May 11, 2022

Score ___________

Show all work to receive full credit. For each problem, place your final answer in the box provided. Each problem is worth 5 points–up to 2 points for the answer and up to 3 points for the supporting work or explanation. Derivatives need not be simplified.

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1. Determine the limit. Show analytically (not with a graph or table) how you got your answer.

$$\lim_{x \to 0} \frac{1 + \tan x}{x^2(x-7)}$$

2. The function $f(x) = \dfrac{x^2 + x}{x^2 - 2x}$ has two discontinuities. Find and classify them.

3. Use a table of numerical values to estimate the limit: $\displaystyle\lim_{x\to 1^+}\left(\frac{3}{\ln x}-\frac{2}{x-1}\right)$

4. Determine the limit. Use algebraic techniques (not a graph, table, or L'Hôpital's rule) to show how you got your answer.

$$\lim_{x\to 16}\left(\frac{16-x}{4-\sqrt{x}}\right)$$

5. Find $\dfrac{dy}{dx}$ if $y=\dfrac{\sqrt{x}+5x^3}{\cos x}$.

6. Let $f(x) = x^2 - x$. Write $f'(x)$ in the box, then use the limit definition of derivative to obtain your answer.

7. Compute $g'(1/2)$ if $g(x) = 1 + 5\cot(\pi x^2)$.

8. A potato is launched vertically upward with an initial velocity of 100 ft/s from a potato gun at the top of an 85-foot-tall building. The height in feet of the potato after t seconds (measured from the ground) is given by $s(t) = -16t^2 + 100t + 85$. Use calculus to find the maximum height of the potato.

9. Let $h(x) = \cos^{-1}(3x) - \tan^{-1}(x^3)$. Determine $h'(x)$.

10. Compute $f'(0)$ if $f(x) = e^{\sin(2x)}$.

11. The gamma function, $\Gamma(x)$, is a special function that arises in many applications. Use the values,

$$\Gamma(1) = 1 \qquad \text{and} \qquad \Gamma'(1) = -0.577215665,$$

to find the linearization for Γ at $x = 1$. Then use your linearization to approximate $\Gamma(1.05)$.

12. Evaluate the limit: $\displaystyle \lim_{x \to 0} \frac{\tan^{-1} x}{\pi x}$

13. Find the critical number(s) of $f(x) = \dfrac{1}{2x^3 - 3x^2}$.

14. The function f has critical numbers $x = 0$, $x = 1/2$, and $x = 1$. Furthermore, its second derivative is given by $f''(x) = 12x^2 - 12x + 2$. Use the second derivative to determine whether the critical numbers give relative maxima or minima.

15. Use calculus techniques to find the absolute extreme values of $f(x) = (x + 2)^2 e^{-x}$ on $[-3, 1]$. (Remember that $e^{-x} > 0$ for all x.)

16. Evaluate the indefinite integral: $\displaystyle\int \left(\frac{2}{x} + e^x - \frac{1}{x^2}\right) dx$

17. Let $f(x) = \ln x$. Use 5 subintervals of equal length and subinterval midpoints (for the c_k's) to compute a Riemann sum for f on $[1, 2]$.

18. Evaluate the definite integral: $\int_1^9 \sqrt{x}\, dx$

19. Use a definite integral to find the area of the bounded region between the graph of $y = 4 - x^2$ and the x-axis.

20. Use a u-substitution to evaluate the definite integral: $\int_0^1 3x(x^2 + 1)^4\, dx$