

# Math 131 - Final Exam

May 11, 2022

Name \_\_\_\_\_

Score \_\_\_\_\_

**Show all work to receive full credit.** For each problem, place your final answer in the box provided. Each problem is worth 5 points—up to 2 points for the answer and up to 3 points for the supporting work or explanation. Derivatives need not be simplified.

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1. Determine the limit. Show analytically (not with a graph or table) how you got your answer.

$$\lim_{x \rightarrow 0} \frac{1 + \tan x}{x^2(x - 7)}$$

2. The function  $f(x) = \frac{x^2 + x}{x^2 - 2x}$  has two discontinuities. Find and classify them.

3. Use a table of numerical values to estimate the limit:  $\lim_{x \rightarrow 1^+} \left( \frac{3}{\ln x} - \frac{2}{x-1} \right)$

4. Determine the limit. Use algebraic techniques (not a graph, table, or L'Hôpital's rule) to show how you got your answer.

$$\lim_{x \rightarrow 16} \left( \frac{16 - x}{4 - \sqrt{x}} \right)$$

5. Find  $\frac{dy}{dx}$  if  $y = \frac{\sqrt{x} + 5x^3}{\cos x}$ .

6. Let  $f(x) = x^2 - x$ . Write  $f'(x)$  in the box, then use the limit definition of derivative to obtain your answer.

7. Compute  $g'(1/2)$  if  $g(x) = 1 + 5 \cot(\pi x^2)$ .

8. A potato is launched vertically upward with an initial velocity of 100 ft/s from a potato gun at the top of an 85-foot-tall building. The height in feet of the potato after  $t$  seconds (measured from the ground) is given by  $s(t) = -16t^2 + 100t + 85$ . Use calculus to find the maximum height of the potato.

9. Let  $h(x) = \cos^{-1}(3x) - \tan^{-1}(x^3)$ . Determine  $h'(x)$ .

10. Compute  $f'(0)$  if  $f(x) = e^{\sin(2x)}$ .

11. The gamma function,  $\Gamma(x)$ , is a special function that arises in many applications. Use the values,

$$\Gamma(1) = 1 \quad \text{and} \quad \Gamma'(1) = -0.577215665,$$

to find the linearization for  $\Gamma$  at  $x = 1$ . Then use your linearization to approximate  $\Gamma(1.05)$ .

12. Evaluate the limit:  $\lim_{x \rightarrow 0} \frac{\tan^{-1} x}{\pi x}$

13. Find the critical number(s) of  $f(x) = \frac{1}{2x^3 - 3x^2}$ .

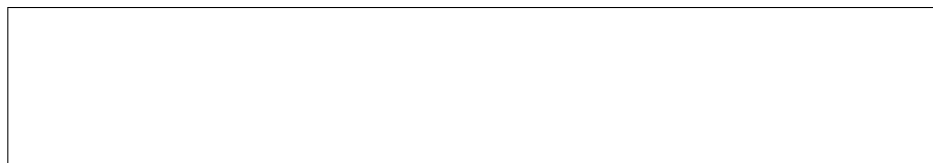
14. The function  $f$  has critical numbers  $x = 0$ ,  $x = 1/2$ , and  $x = 1$ . Furthermore, its second derivative is given by  $f''(x) = 12x^2 - 12x + 2$ . Use the second derivative to determine whether the critical numbers give relative maxima or minima.

15. Use calculus techniques to find the absolute extreme values of  $f(x) = (x + 2)^2 e^{-x}$  on  $[-3, 1]$ . (Remember that  $e^{-x} > 0$  for all  $x$ .)

16. Evaluate the indefinite integral:  $\int \left( \frac{2}{x} + e^x - \frac{1}{x^2} \right) dx$



17. Let  $f(x) = \ln x$ . Use 5 subintervals of equal length and subinterval midpoints (for the  $c_k$ 's) to compute a Riemann sum for  $f$  on  $[1, 2]$ .



18. Evaluate the definite integral:  $\int_1^9 \sqrt{x} dx$

19. Use a definite integral to find the area of the bounded region between the graph of  $y = 4 - x^2$  and the  $x$ -axis.

20. Use a  $u$ -substitution to evaluate the definite integral:  $\int_0^1 3x(x^2 + 1)^4 dx$