Math 131 - Quiz 10

April 19, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (7 points) Evaluate each limit.

(a) $\lim_{x\to 0} \frac{\sin 5x}{\tan 9x}$ $= \lim_{x\to 0} \frac{5\cos 5x}{9\sec^2 9x} = \frac{5\cdot 1}{9\cdot 1} = \boxed{\frac{5}{9}}$ L'HÔPITAL'S RULE

OR WEEK 2 APPROACH...

$$\lim_{x\to 0} \frac{\sin 5x}{\tan 9x} = \lim_{x\to 0} \left(\frac{5}{9}\cdot \frac{\sin 5x}{5x}\cdot \frac{9x}{\sin 9x}\cdot \frac{\cos 9x}{1}\right) = \frac{5}{9}\cdot 1\cdot 1\cdot 1 = \frac{5}{9}$$

(b) $\lim_{x\to\infty} e^{-x}\sqrt{x}$

$$= \lim_{x\to\infty} \frac{\sqrt{x}}{e^x} = \lim_{x\to\infty} \frac{1}{2\sqrt{x}\,e^x} = \boxed{0}$$

(c) $\lim_{x\to 2^+} \left(\frac{8}{x^2-4} - \frac{x}{x-2}\right)$ $= \lim_{x\to 2^+} \left(\frac{8}{x^2-4} - \frac{x(x+2)}{x^2-4}\right)$

$$= \lim_{x\to 2^+} \frac{8-x^2-2x}{x^2-4} = \lim_{x\to 2^+} \frac{-(x-2)(x+4)}{(x-2)(x+2)} = \frac{-6}{4} = \boxed{-\frac{3}{2}}$$

OR USE L'HÔPITAL'S RULE.

2. (3 points) Use Newton's method to approximate the solution of $\sin x = x - 1$.

$f(x) = 1 - x + \sin x$

$f'(x) = -1 + \cos x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

From graph of f, $x_0 = 2$ looks like a good starting guess.

$x_0 = 2$

$x_1 = 1.935951152$

$x_2 = 1.934563874$

$x_3 = 1.934563211$

x_4 same as x_3

Solution ≈ 1.93595