Math 131 - Quiz 12

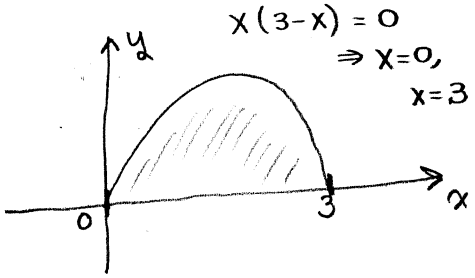
May 3, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Use a definite integral to find the area of the bounded region above the x-axis and below the graph of $y = 3x - x^2$.



$x(3-x) = 0$

$\Rightarrow x = 0,\ x = 3$

$y = x(3-x)$

$$\text{AREA} = \int_0^3 (3x - x^2)\,dx$$

$$= \frac{3}{2}x^2 - \frac{1}{3}x^3 \Big|_0^3$$

$$= \frac{27}{2} - 9 = \boxed{\frac{9}{2}}$$

2. (4 points) Use the fundamental theorem of calculus to evaluate each definite integral.

(a) $\displaystyle\int_0^{\pi} \sin x\,dx$ $= -\cos x \Big|_0^{\pi} = \cos x \Big|_{\pi}^{0} =$

$\cos 0 - \cos(\pi) = 1 - (-1) = \boxed{2}$

(b) $\displaystyle\int_1^2 \left(\frac{1}{x} - e^x\right) dx$ $= \ln|x| - e^x \Big|_1^2 = \left(\ln(2) - e^2\right) - \left(\ln(1) - e\right)$

$= \boxed{e - e^2 + \ln(2)}$

≈ -3.9776

3. (1 point) Let $F(x) = \displaystyle\int_x^{\pi/4} t\tan t\,dt$. Determine $F'(x)$.

$$F(x) = -\int_{\pi/4}^{x} t\tan t\,dt$$

$$\boxed{F'(x) = -x\tan x}$$