Math 131 - Test 1

February 6, 2023

Name key Score ________

Show all work to receive full credit. Supply explanations where necessary. You may get partial credit on multiple choice problems if you supply correct work or explanations.

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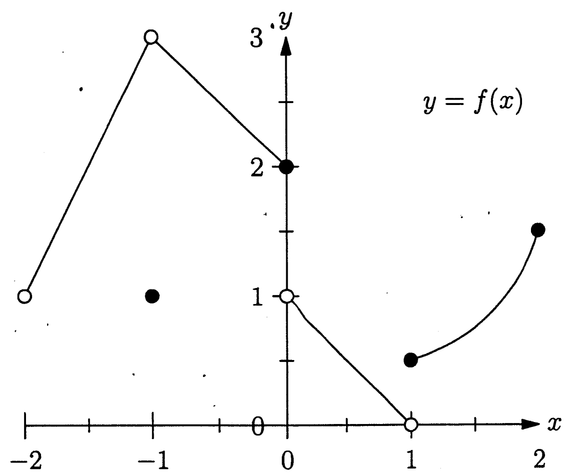
1. (10 points) Referring to the graph shown below, determine each of the following or explain why it does not exist.



(a) $\lim_{x \to -1} f(x) = 3$

(b) $\lim_{x \to -2^+} f(x) = 1$

(c) $\lim_{x \to 1} f(x)$ DNE $\quad \lim_{x \to 1^+} f(x) = 0.5 \neq \lim_{x \to 1^-} f(x) = 0$

(d) $\lim_{x \to 0^-} f(x) = 2$

(e) $\lim_{x \to 2} f(x)$ DNE $\quad f$ IS NOT DEFINED FOR $X > 2$.

A TWO-SIDED LIMIT AT $X = 2$ MAKES NO SENSE.

2. (2 points) Referring to the function above, find a point at which f is defined but is not continuous. State the type of discontinuity at that point.

$X = -1$ REMOVABLE

$X = 0$ JUMP

$X = 1$ JUMP

3. (2 points) Suppose you were asked to use a table of values to estimate $\lim_{x \to 5} f(x)$. Which list of x-values shown below would be best for your table?

(a) $x = 5.01, 5.001, 5.0001, 5, 4.99, 4.999, 4.9999$

(b) $x = 4.0, 4.5, 4.75, 5.0, 5.25, 5.5, 6.0$

(c) $x = 5.01, 5.001, 5.0001, 4.99, 4.999, 4.9999$

(d) $x = 4.9, 4.99, 4.999, 4.9999, 5.1$

4. (2 points) Suppose $\lim_{x \to 1} f(x) = 8$. Which one of these statements is true?

(a) The function f must be defined at $x = 1$.

(b) $f(1) = 8$

(c) The domain of f cannot include the number 1.

(d) The domain of f must include some numbers less than 1.

5. (2 points) Explain why this limit fails to exist: $\lim_{x \to -1} \frac{x+1}{|x+1|}$.

$\lim_{x \to -1^-} \frac{x+1}{|x+1|} = -1$

(a) Direct substitution results in division by zero.

$\lim_{x \to -1^+} \frac{x+1}{|x+1|} = +1$

(b) The limit from the left does not equal the limit from the right.

(c) The function values grow without bound as the limit point is approached.

(d) The function is not defined on both sides of the limit point.

6. (2 points) Explain why this limit fails to exist.

$$\lim_{x \to 0} f(x) \quad \text{where} \quad f(x) = \begin{cases} (x-3)/x, & x < 0 \\ \cos(x)/x, & x > 0 \end{cases}$$

(a) The function is not defined at $x = 0$.

(b) The limit from the left does not equal the limit from the right.

$\frac{\text{NONZERO}}{\text{ZERO}}$ FORMS

(c) The function values oscillate as the limit point is approached.

(d) The function values grow without bound as the limit point is approached.

7. (2 points) Explain why this limit fails to exist: $\lim_{x \to 0} \cos\left(\frac{1}{x^2}\right)$.

(a) Direct substitution results in division by zero.

(b) The function values grow without bound as the limit point is approached.

(c) The function values oscillate as the limit point is approached.

(d) The limit from the left does not equal the limit from the right.

8. (24 points) Determine each limit analytically, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE.

(a) $\lim_{x\to 0} \frac{(x-2)^2-4}{x}$ 0/0 MORE WORK.

$$\lim_{x\to 0} \frac{x^2-4x+4-4}{x} = \lim_{x\to 0} \frac{x^2-4x}{x} = \lim_{x\to 0}(x-4) = \boxed{-4}$$

(b) $\lim_{k\to 4} \frac{\sqrt{k}-2}{2k-8}$ 0/0 MORE WORK.

$$\lim_{k\to 4} \frac{\sqrt{k}-2}{2(k-4)} \cdot \frac{\sqrt{k}+2}{\sqrt{k}+2} = \lim_{k\to 4} \frac{\cancel{k-4}}{2(\cancel{k-4})(\sqrt{k}+2)} = \frac{1}{2\cdot 4} = \boxed{\frac{1}{8}}$$

(c) $\lim_{x\to 4^-} \left(\frac{x-4}{x^2-8x+16}\right)$ 0/0 MORE WORK.

1/0 SOME KIND OF INF. LIMIT.

$$\lim_{x\to 4^-} \frac{\cancel{x-4}}{(\cancel{x-4})(x-4)} = \lim_{x\to 4^-} \frac{1}{x-4}$$

FROM LEFT OF $x=4$

$$\frac{1}{x-4} = \frac{+}{-} = - \Rightarrow \boxed{\text{LIMIT IS } -\infty.}$$

(d) $\lim_{z\to 0} \left(\frac{3\tan 2z}{4z}\right)$ 0/0 MORE WORK.

$$= \lim_{z\to 0} \frac{3}{2}\,\frac{\tan 2z}{2z} = \frac{3}{2}\lim_{z\to 0} \frac{\sin 2z}{2z\cos 2z}$$

$$= \frac{3}{2}\lim_{x\to 0}\left(\frac{\sin 2z}{2z}\cdot\frac{1}{\cos 2z}\right) = \frac{3}{2}(1)(1) = \boxed{\frac{3}{2}}$$

9. (6 points) Use a table of numerical values to approximate the following limit. Your table must show function values at four or more points.

$$\lim_{x \to 0+} \frac{2^x - 1}{3x}$$

$x \to 0^+$	$\frac{2^x - 1}{3x}$
0.1	0.239245
0.01	0.231852
0.001	0.231129
0.0001	0.231057
0.00001	0.231050

IT LOOKS LIKE $\lim_{x \to 0^+} \frac{2^x - 1}{3x} \approx 0.231$

THE EXACT LIMIT IS $\frac{\ln 2}{3}$.

10. (6 points) Determine all values of k for which g is continuous everywhere.

$$g(x) = \begin{cases} x^2 + 4x - 8, & x < k \\ 1 + 4x, & x \geq k \end{cases}$$

EACH PIECE OF g IS CONTINUOUS EVERYWHERE.

WE NEED TO FOCUS ON WHERE THE PIECES "BREAK" AT $X = k$.

WE NEED $\lim_{x \to k^-} g(x) = \lim_{x \to k^+} g(x) = g(k)$ FOR CONTINUITY AT $X = k$.

$$k^2 + 4k - 8 = 1 + 4k = 1 + 4k$$

$$k^2 - 9 = 0 \Rightarrow \boxed{k = \pm 3}$$

11. (9 points) Consider the rational function $R(x) = \frac{5x^2 - 5}{x^2 - 8x + 7}$. Find all points at which R is discontinuous, and state whether each discontinuity is removable or nonremovable. Also tell where the graph of R has vertical asymptotes.

$$R(x) = \frac{5(x+1)(x-1)}{(x-1)(x-7)}$$

DENOM ZERO WHEN $X = 1$, $X = 7$

DISCONTS AT $X = 7$ AND $X = 1$

SINCE $\lim_{x \to 1} R(x) = \lim_{x \to 1} \frac{5(x+1)}{x-7} = \frac{10}{-6}$,

$X = 1$ IS REMOVABLE.

SINCE $X = 7$ GIVES A NONZERO OVER ZERO FORM, $X = 7$ IS NONREMOVABLE (INFINITE).

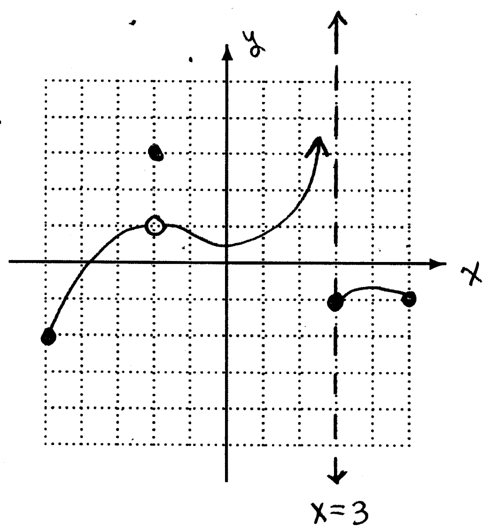
$\Downarrow$

$\boxed{X = 7 \text{ IS THE ONLY V.A.}}$

14. (8 points) Sketch the graph of a function f such that

- √ f is defined for all real numbers between -5 and 5,
- √ $f(-2) = 3$,
- √ f has a removable discontinuity at $x = -2$,
- √ $\lim_{x \to 3^-} f(x) = \infty$, and
- √ $\lim_{x \to 3^+} f(x) = -1$.

THERE ARE LOTS OF POSSIBLE ANSWERS!



15. (5 points) Given that $-x^4 \le x^4 \sin \frac{1}{x^2} \le x^4$ when $x \neq 0$, compute $\lim_{x \to 0} x^4 \sin \frac{1}{x^2}$.

Explain and state the name of the theorem you used.

$\boxed{\text{SQUEEZE THEOREM}}$

$$\lim_{X \to 0} (-X^4) = 0 = \lim_{X \to 0} (X^4) \Rightarrow \lim_{X \to 0} X^4 \sin \frac{1}{X^2} = 0$$

16. (5 points) Each row of the table below gives some information about a function f. Fill in each blank entry with an appropriate word or number. In some cases there may be more than one correct answer.

Continuous at $x = 2$	$f(2)$	$\lim_{x \to 2^-} f(x)$	$\lim_{x \to 2^+} f(x)$
Yes	5	5	5
No	7	Any # $\neq$ 7	7
No	Any # $\neq$ -1	-1	-1
YES	2	2	2
Yes	1	1	1

15. (9 points) For each part of this problem, assume that $\lim_{x\to 2} f(x) = 3$ and $\lim_{x\to 2} g(x)$ exists. Show work or explain your reasoning.

(a) Evaluate $\lim_{x\to 2} \left[x^2 f(x) + g(x)\sin(\pi x)\right]$.

$$\lim_{x\to 2} x^2 \cdot \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) \cdot \lim_{x\to 2} \sin(\pi x)$$

$$= 4\cdot 3 + \left[\lim_{x\to 2} g(x)\right]\cdot 0 = \boxed{12}$$

(b) Find $\lim_{x\to 2} g(x)$ if $\lim_{x\to 2} \dfrac{4}{(g(x))^2} = 16$.

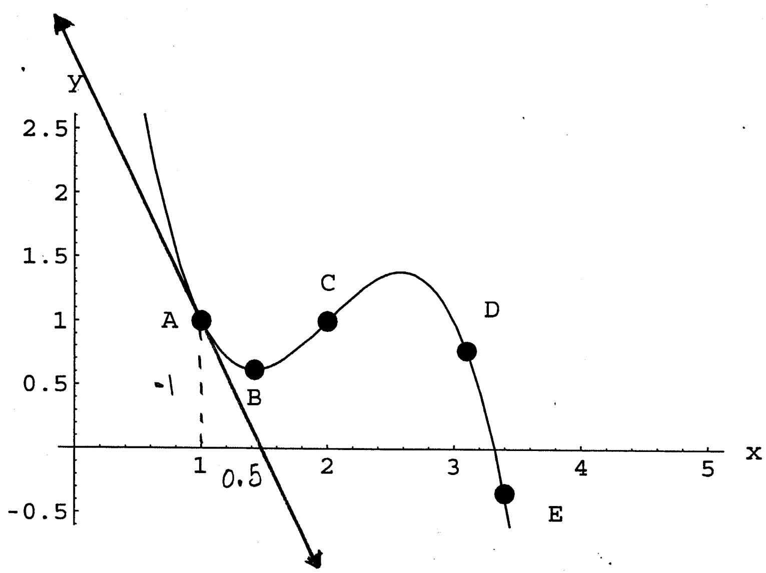
$$\frac{4}{\left[\lim_{x\to 2} g(x)\right]^2} = 16 \Rightarrow \left[\lim_{x\to 2} g(x)\right]^2 = \frac{1}{4} \Rightarrow \boxed{\lim_{x\to 2} g(x) = \pm\frac{1}{2}}$$

(c) Find $\lim_{x\to 2} g(x)$ if $\underbrace{\lim_{x\to 2} \sqrt{f(x)\,g(x)}}$ does not exist. (There is more than one possible answer.)

$$\sqrt{\lim_{x\to 2} f(x) \cdot \lim_{x\to 2} g(x)} = \sqrt{3 \cdot \underbrace{\lim_{x\to 2} g(x)}_{\text{Must be a Neg \#}}}$$

$\boxed{\lim_{x\to 2} g(x) = \text{Any } k < 0}$!

16. (6 points) The graph of $y = f(x)$ is shown below. Use the graph to solve each part of this problem.



(a) Estimate the derivative of f at the point labeled B. Explain your reasoning.

Tangent line is horizontal $\Rightarrow$ $\boxed{\text{Derivative} = 0}$

(b) At which of the labeled points is the derivative the greatest? Explain.

At C. This is the only labeled point at which the derivative is positive.

(c) Sketch the tangent line at the point labeled A. Use rise over run to approximate its slope.

See tangent line above.

$$\text{Slope} \approx \frac{-1}{0.5} = \boxed{-2}$$