

# Math 131 - Test 1

February 8, 2023

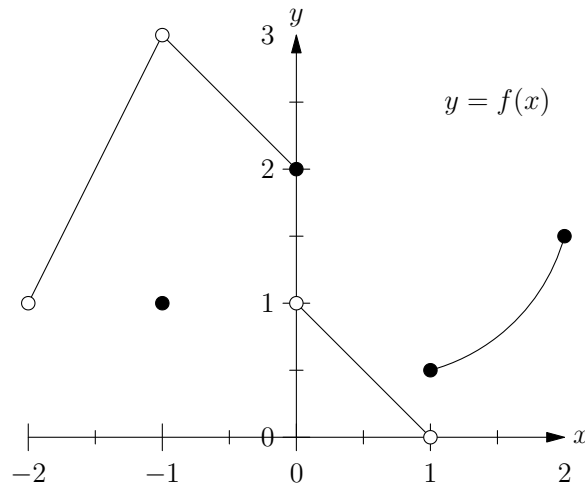
Name \_\_\_\_\_

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations where necessary. You may get partial credit on multiple choice problems if you supply correct work or explanations.

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1. (10 points) Referring to the graph shown below, determine each of the following or explain why it does not exist.



(a)  $\lim_{x \rightarrow -1} f(x)$

(b)  $\lim_{x \rightarrow -2^+} f(x)$

(c)  $\lim_{x \rightarrow 1} f(x)$

(d)  $\lim_{x \rightarrow 0^-} f(x)$

(e)  $\lim_{x \rightarrow 2} f(x)$

2. (2 points) Referring to the function above, find a point at which  $f$  is defined but is not continuous. State the type of discontinuity at that point.

3. (2 points) Suppose you were asked to use a table of values to estimate  $\lim_{x \rightarrow 5} f(x)$ . Which list of  $x$ -values shown below would be best for your table?
- (a)  $x = 5.01, 5.001, 5.0001, 5, 4.99, 4.999, 4.9999$
  - (b)  $x = 4.0, 4.5, 4.75, 5.0, 5.25, 5.5, 6.0$
  - (c)  $x = 5.01, 5.001, 5.0001, 4.99, 4.999, 4.9999$
  - (d)  $x = 4.9, 4.99, 4.999, 4.9999, 5.1$
4. (2 points) Suppose  $\lim_{x \rightarrow 1} f(x) = 8$ . Which one of these statements is true?
- (a) The function  $f$  must be defined at  $x = 1$ .
  - (b)  $f(1) = 8$
  - (c) The domain of  $f$  cannot include the number 1.
  - (d) The domain of  $f$  must include some numbers less than 1.
5. (2 points) Explain why this limit fails to exist:  $\lim_{x \rightarrow -1} \frac{x+1}{|x+1|}$ .
- (a) Direct substitution results in division by zero.
  - (b) The limit from the left does not equal the limit from the right.
  - (c) The function values grow without bound as the limit point is approached.
  - (d) The function is not defined on both sides of the limit point.
6. (2 points) Explain why this limit fails to exist.
- $$\lim_{x \rightarrow 0} f(x) \quad \text{where} \quad f(x) = \begin{cases} (x-3)/x, & x < 0 \\ \cos(x)/x, & x > 0 \end{cases}$$
- (a) The function is not defined at  $x = 0$ .
  - (b) The limit from the left does not equal the limit from the right.
  - (c) The function values oscillate as the limit point is approached.
  - (d) The function values grow without bound as the limit point is approached.
7. (2 points) Explain why this limit fails to exist:  $\lim_{x \rightarrow 0} \cos\left(\frac{1}{x^2}\right)$ .
- (a) Direct substitution results in division by zero.
  - (b) The function values grow without bound as the limit point is approached.
  - (c) The function values oscillate as the limit point is approached.
  - (d) The limit from the left does not equal the limit from the right.

8. (24 points) Determine each limit analytically, or explain why the limit does not exist. You may need to use  $+\infty$ ,  $-\infty$ , or DNE.

(a)  $\lim_{x \rightarrow 0} \frac{(x-2)^2 - 4}{x}$

(b)  $\lim_{k \rightarrow 4} \frac{\sqrt{k} - 2}{2k - 8}$

(c)  $\lim_{x \rightarrow 4^-} \left( \frac{x-4}{x^2 - 8x + 16} \right)$

(d)  $\lim_{z \rightarrow 0} \left( \frac{3 \tan 2z}{4z} \right)$

9. (6 points) Use a table of numerical values to approximate the following limit. Your table must show function values at four or more points.

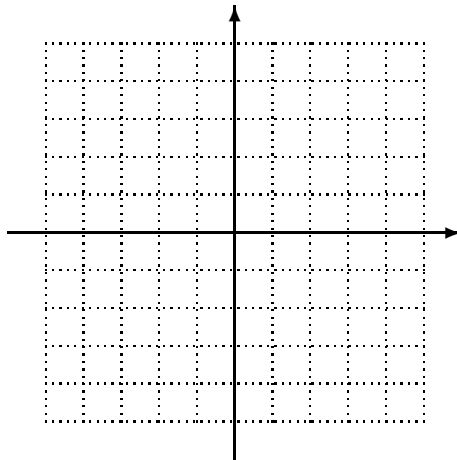
$$\lim_{x \rightarrow 0^+} \frac{2^x - 1}{3x}$$

10. (6 points) Determine all values of  $k$  for which  $g$  is continuous everywhere.

$$g(x) = \begin{cases} x^2 + 4x - 8, & x < k \\ 1 + 4x, & x \geq k \end{cases}$$

11. (9 points) Consider the rational function  $R(x) = \frac{5x^2 - 5}{x^2 - 8x + 7}$ . Find all points at which  $R$  is discontinuous, and state whether each discontinuity is removable or nonremovable. Also tell where the graph of  $R$  has vertical asymptotes.

12. (8 points) Sketch the graph of a function  $f$  such that
- $f$  is defined for all real numbers between  $-5$  and  $5$ ,
  - $f(-2) = 3$ ,
  - $f$  has a removable discontinuity at  $x = -2$ ,
  - $\lim_{x \rightarrow 3^-} f(x) = \infty$ , and
  - $\lim_{x \rightarrow 3^+} f(x) = -1$ .



13. (5 points) Given that  $-x^4 \leq x^4 \sin \frac{1}{x^2} \leq x^4$  when  $x \neq 0$ , compute  $\lim_{x \rightarrow 0} x^4 \sin \frac{1}{x^2}$ .
- Explain, and state the name of the theorem you used.

14. (5 points) Each row of the table below gives some information about a function  $f$ . Fill in each blank entry with an appropriate word or number. In some cases there may be more than one correct answer.

Continuous at $x = 2$	$f(2)$	$\lim_{x \rightarrow 2^-} f(x)$	$\lim_{x \rightarrow 2^+} f(x)$
Yes	5	5	
No	7		7
No		-1	-1
	2	2	2
Yes		1	

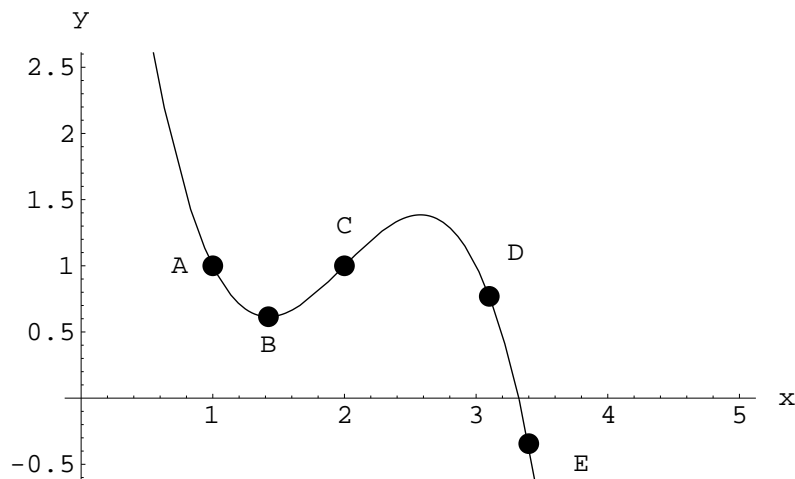
15. (9 points) For each part of this problem, assume that  $\lim_{x \rightarrow 2} f(x) = 3$  and  $\lim_{x \rightarrow 2} g(x)$  exists. Show work or explain your reasoning.

(a) Evaluate  $\lim_{x \rightarrow 2} [x^2 f(x) + g(x) \sin(\pi x)]$ .

(b) Find  $\lim_{x \rightarrow 2} g(x)$  if  $\lim_{x \rightarrow 2} \frac{4}{(g(x))^2} = 16$ .

(c) Find  $\lim_{x \rightarrow 2} g(x)$  if  $\lim_{x \rightarrow 2} \sqrt{f(x)g(x)}$  does not exist. (There is more than one possible answer.)

16. (6 points) The graph of  $y = f(x)$  is shown below. Use the graph to solve each part of this problem.



- (a) Estimate the derivative of  $f$  at the point labeled B. Explain your reasoning.
- (b) At which of the labeled points is the derivative the greatest? Explain.
- (c) Sketch the tangent line at the point labeled A. Use rise over run to approximate its slope.