

Math 131 - Quiz 10

April 10, 2023

Name Key

Score _____

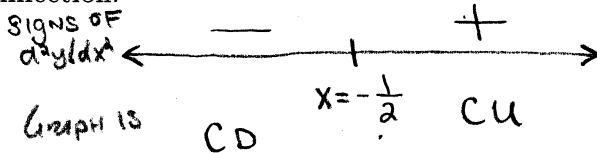
Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Find open intervals on which the graph of $y = 2x^3 + 3x^2 + 1$ is concave up/down. Identify all points of inflection.

$$\frac{dy}{dx} = 6x^2 + 6x$$

$$\frac{d^2y}{dx^2} = 12x + 6 = 0$$

$$\Downarrow \\ x = -\frac{1}{2}$$



Graph is concave down on $(-\infty, -\frac{1}{2})$ and concave up on $(-\frac{1}{2}, \infty)$.

$(-\frac{1}{2}, \frac{3}{2})$ is the only inflection pt.

2. (3 points) Use the 2nd derivative to determine whether the graph of $y = x^3 + \sin(10x)$ is concave up or concave down at the point where $x = 0.65$.

$$\frac{dy}{dx} = 3x^2 + 10\cos(10x)$$

$$\frac{d^2y}{dx^2} \Big|_{x=0.65} \approx -17.612$$

≈ -17.612

Graph is Concave Down.

$$\frac{d^2y}{dx^2} = 6x - 100\sin(10x)$$

3. (3 points) Determine each limit.

$$(a) \lim_{x \rightarrow -\infty} \frac{8 + 2x - 9x^2}{1 + 3x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x \rightarrow -\infty} \frac{\frac{8}{x^2} + \frac{2}{x} - 9}{\frac{1}{x^2} + 3} = \frac{-9}{3} = \boxed{-3}$$

$$(b) \lim_{x \rightarrow \infty} \frac{\sqrt{x^3}}{x^2 - x - 1} = \lim_{x \rightarrow \infty} \frac{x^{3/2}}{x^2 - x - 1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{1}{x^{1/2} \left(1 - \frac{1}{x} - \frac{1}{x^2}\right)} = \frac{0}{1} = \boxed{0}$$