Math 131 - Quiz 3

January 30, 2023

Name key

Score ______

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) For each part of this problem, assume that $\lim_{x\to 2} f(x) = 3$, $\lim_{x\to 2} g(x) = 7$, and $\lim_{x\to 2} h(x)$ exists.

   (a) Find $\lim_{x\to 2} h(x)$ if $\lim_{x\to 2} \frac{g(x)}{h(x)} = \frac{1}{2}$.

   $$\frac{\lim_{x\to 2} g(x)}{\lim_{x\to 2} h(x)} = \frac{7}{\lim_{x\to 2} h(x)} = \frac{1}{2} \Rightarrow \boxed{\lim_{x\to 2} h(x) = 14}$$

   (b) Find $\lim_{x\to 2} h(x)$ if $\lim_{x\to 2} \frac{f(x)}{h(x)}$ does not exist.

   $$\frac{\lim_{x\to 2} f(x)}{\lim_{x\to 2} h(x)} = \frac{3}{\lim_{x\to 2} h(x)}$$

   THE ONLY WAY THIS FRACTION DNE IS IF $\boxed{\lim_{x\to 2} h(x) = 0}$.

2. (3 points) Explain why direct substitution cannot be used to evaluate the limit. Then use a different approach to find the limit.

   $0/0$. MORE WORK!

   $$\lim_{x\to 5} \left(\frac{x^2 - 3x - 10}{x^2 + x - 30} \right)$$

   DIRECT SUBS GIVES A ZERO DENOM

   $\Rightarrow$ NOT GOING TO GET LIMIT THAT WAY!

   $$= \lim_{x\to 5} \frac{\cancel{(x-5)}(x+2)}{\cancel{(x-5)}(x+6)} = \boxed{\frac{7}{11}}$$

Turn over.

3. (2 points) Evaluate the limit: $\lim_{y\to 2} \frac{2y-4}{\sqrt{y}-\sqrt{2}}$

% More work!

$$\lim_{y\to 2} \frac{2y-4}{\sqrt{y}-\sqrt{2}} \cdot \frac{\sqrt{y}+\sqrt{2}}{\sqrt{y}+\sqrt{2}} = \lim_{y\to 2} \frac{(2y-4)(\sqrt{y}+\sqrt{2})}{y-2}$$

$$= \lim_{y\to 2} \frac{2\cancel{(y-2)}(\sqrt{y}+\sqrt{2})}{\cancel{y-2}} = 2(\sqrt{2}+\sqrt{2})$$

$$= \boxed{4\sqrt{2} \approx 5.657.}$$

4. (2 points) Evaluate $\lim_{x\to 2^-} f(x)$, where $f(x) = \begin{cases} 2x^3 + \cos(\pi x), & -3 \le x < 2 \\ x\sin(x), & x > 2 \end{cases}$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \left(2x^3 + \cos(\pi x)\right)$$

$$= 2(2)^3 + \cos(2\pi)$$

$$= 16 + 1 = \boxed{17}$$