Math 131 - Quiz 4

February 13, 2023

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due February 20. You must work individually on this quiz.

1. (4 points) Use the limit definition of derivative to determine $f'(x)$ when $f(x) = 5 + 6x - 2x^2$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{[5+6(x+h)-2(x+h)^2] - [5+6x-2x^2]}{h}$$

$$= \lim_{h\to 0} \frac{5+6x+6h-2x^2-4xh-2h^2-5-6x+2x^2}{h}$$

$$= \lim_{h\to 0} \frac{6h-4xh-2h^2}{h} = \lim_{h\to 0} \frac{h(6-4x-2h)}{h}$$

$$= \lim_{h\to 0} (6-4x-2h) = \boxed{6-4x}$$

2. (2 points) Suppose $F(x) = (x+2)e^{x^2}$. Later, we will learn how to show that $F'(x) = (2x^2+4x+1)e^{x^2}$. For now, just use the given information to find an equation of the line tangent to the graph of F at the point where $x = 0$.

POINT: $x = 0 \Rightarrow y = F(0) = 2$

$(0,2)$

SLOPE $= F'(0) = 1$

TANGENT LINE:

$y - 2 = 1(x - 0)$

OR

$\boxed{y = x + 2}$

Turn over.

3. (4 points) Use the limit definition of the derivative to determine $g'(x)$ when $g(x) = \sqrt{2x}$.

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{\sqrt{2x+2h} - \sqrt{2x}}{h} \cdot \frac{\sqrt{2x+2h} + \sqrt{2x}}{\sqrt{2x+2h} + \sqrt{2x}}$$

$$= \lim_{h \to 0} \frac{2x + 2h - 2x}{h\left(\sqrt{2x+2h} + \sqrt{2x}\right)} = \lim_{h \to 0} \frac{2h}{h\left(\sqrt{2x+2h} + \sqrt{2x}\right)}$$

$$= \lim_{h \to 0} \frac{2}{\sqrt{2x+2h} + \sqrt{2x}} = \frac{2}{\sqrt{2x} + \sqrt{2x}}$$

$$= \boxed{\frac{1}{\sqrt{2x}}}$$