Math 131 - Quiz 7

March 20, 2023

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due March 27.

1. (3 points) For $x \geq 1$, let $g(x) = x^2 - 2x + 5$. The function g has an inverse.

   (a) Determine the value of $g^{-1}(8)$.

$$g^{-1}(8) = \omega \Leftrightarrow g(\omega) = 8 \Leftrightarrow \omega^2 - 2\omega + 5 = 8 \Leftrightarrow \omega^2 - 2\omega - 3 = 0$$

$$\omega^2 - 2\omega - 3 = (\omega - 3)(\omega + 1) = 0 \qquad \boxed{g^{-1}(8) = 3}$$

$$\omega = 3 \text{ or } \cancel{\omega = -1}$$

   (b) Now find $(g^{-1})'(8)$.

$$g'(x) = 2x - 2$$

$$(g^{-1})'(8) = \frac{1}{g'(g^{-1}(8))} = \frac{1}{g'(3)} = \frac{1}{2(3) - 2} = \boxed{\frac{1}{4}}$$

2. (4 points) Evaluate each derivative.

   (a) $\frac{d}{dx} \sin^{-1}(\pi x^2) = \frac{1}{\sqrt{1 - (\pi x^2)^2}} \cdot \frac{d}{dx} \pi x^2 = \boxed{\frac{2\pi x}{\sqrt{1 - \pi^2 x^4}}}$

   (b) $\frac{d}{dx}[e^{-3x} \overbrace{\ln(x^2)}^{2\ln x}] = \frac{d}{dx}\left[e^{-3x} \cdot 2\ln x\right]$

$$= -3e^{-3x} \cdot 2\ln x + \frac{2e^{-3x}}{x}$$

$$= \boxed{e^{-3x}\left[-6\ln x + \frac{2}{x}\right]}$$

Turn over.

3. (3 points) Use logarithmic differentiation to find dy/dx.

$$y = \frac{x^4(x-8)^2}{(x+2)^3(2x+1)}$$

$$\ln y = 4\ln x + 2\ln(x-8) - 3\ln(x+2) - \ln(2x+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{4}{x} + \frac{2}{x-8} - \frac{3}{x+2} - \frac{2}{2x+1}$$

$$\frac{dy}{dx} = (y)\left(\frac{4}{x} + \frac{2}{x-8} - \frac{3}{x+2} - \frac{2}{2x+1}\right)$$

$$\boxed{\frac{dy}{dx} = \left[\frac{x^4(x-8)^2}{(x+2)^3(2x+1)}\right]\left(\frac{4}{x} + \frac{2}{x-8} - \frac{3}{x+2} - \frac{2}{2x+1}\right)}$$