Math 131 - Test 3

April 17, 2023

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) Use logarithmic differentiation to find dy/dx when $x = 3$.

$$y = \frac{x^2(x-2)^5}{x^2+16}$$

$$\ln y = 2\ln x + 5\ln(x-2) - \ln(x^2+16)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \frac{5}{x-2} - \frac{2x}{x^2+16}$$

$$\frac{dy}{dx} = y\left(\frac{2}{x} + \frac{5}{x-2} - \frac{2x}{x^2+16}\right)$$

WHEN $x = 3$, $y = \frac{9}{25}$

AND $\frac{dy}{dx} = \frac{9}{25}\left(\frac{2}{3} + 5 - \frac{6}{25}\right) = \frac{9}{25}\left(\frac{407}{75}\right)$

$$= \boxed{\frac{1221}{625} = 1.9536}$$

2. (4 points) Evaluate the limit: $\displaystyle\lim_{x\to-\infty}\left(\frac{x^4-4x^3+1}{2-2x^2-7x^4}\right) \cdot \frac{\frac{1}{x^4}}{\frac{1}{x^4}}$

$$= \lim_{x\to-\infty} \frac{1 - \frac{4}{x} + \frac{1}{x^4}}{\frac{2}{x^4} - \frac{2}{x^2} - 7} = \frac{1-0-0}{0-0-7}$$

$$= \boxed{-\frac{1}{7}}$$

3. (8 points) Determine each derivative.

(a) $\frac{d}{dx}[x \sin^{-1}(x^2)]$

$$= \sin^{-1}(x^2) + (x)\left(\frac{1}{\sqrt{1-x^4}}\right)(2x)$$

$$= \boxed{\sin^{-1}(x^2) + \frac{2x^2}{\sqrt{1-x^4}}}$$

(b) $\frac{d}{dx}2e^{\tan x}$ $= \boxed{2e^{\mathrm{TAN}\,x} \cdot \sec^2 x}$

4. (6 points) Some values of $f(x)$ and $f'(x)$ are given in the table below.

x	-1	0	1	2
$f(x)$	-0.5737	0.7027	1.1044	11.2234
$f'(x)$	0.0740	0.2082	0.7423	171.4132

(a) Determine the linearization of f at $x = 1$.

$$L(x) = f(1) + f'(1)(x-1)$$

$$\boxed{L(x) = 1.1044 + 0.7423(x-1)}$$

(b) Use the linearization you found above to approximate $f(0.925)$.

$$f(0.925) \approx 1.1044 + 0.7423(0.925-1)$$

$$\approx \boxed{1.0487}$$

5. (6 points) Let $y = \frac{1}{2x+1}$. Use differentials to approximate Δy as x changes from $x = 1$ to $x = 1.25$.

$$\frac{dy}{dx} = \frac{(2x+1)(0) - (1)(2)}{(2x+1)^2} = \frac{-2}{(2x+1)^2}$$

$$\Delta y \approx \frac{-2}{(\underset{\substack{\uparrow \\ x=1}}{2x}+1)^2} \underset{\substack{\uparrow \\ \Delta x = 0.25}}{\Delta x}$$

$$\Delta y \approx -\frac{2}{9}(0.25) = \boxed{-\frac{1}{18} \quad -0.0\overline{5}}$$

6. (8 points) Use calculus techniques to determine the absolute minimum and maximum values of $f(x) = 5x^2 - 6x^{5/3}$ over $[0, 2]$.

$f'(x) = 10x - 10x^{2/3}$

$10x - 10x^{2/3} = 0$

$\Rightarrow x = x^{2/3}$

$x^3 = x^2$

$x = 0, x = 1$

Crit #: $x = 1$

End pts: $x = 0$, $x = 2$

x	$f(x)$	
0	0	
2	0.9512	← Abs max
1	−1	← Abs min

7. (8 points) Evaluate the limit: $\lim_{x \to 0} \left(\frac{e^x - 4x^2 - 2 + e^{-x}}{x^2} \right)$ $0/0$ More work

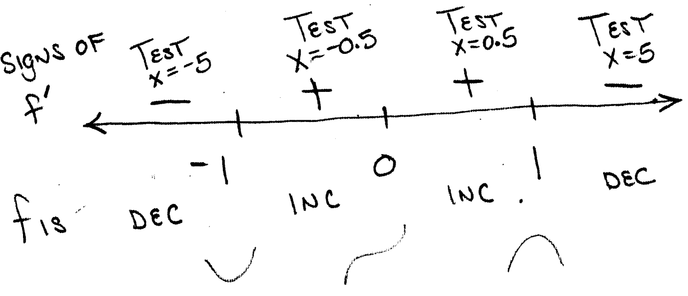
$$= \lim_{x \to 0} \frac{e^x - 8x - e^{-x}}{2x} \quad 0/0 \text{ More work}$$

L'Hopital's Rule twice.

$$= \lim_{x \to 0} \frac{e^x - 8 + e^{-x}}{2} = \frac{-6}{2} = \boxed{-3}$$

8. (8 points) The function $f(x) = 5x^{1/3} - x^{5/3}$ has exactly three critical numbers: $x = -1$, $x = 0$, and $x = 1$. Use calculus techniques to identify all relative extreme values of f.

$$f'(x) = \frac{5}{3}X^{-2/3} - \frac{5}{3}X^{2/3}$$



$\boxed{\begin{array}{l} f(1) = 4 \text{ IS A REL MAX} \\ f(-1) = -4 \text{ IS A REL MIN.} \end{array}}$

9. (8 points) Let $f(x) = x^4 - 2x^3 - 12x^2 + 36x + 2$. Find open intervals on which the graph of f is concave up/down. Also identify all inflection points.

$$f'(x) = 4x^3 - 6x^2 - 24x + 36$$

$$f''(x) = 12x^2 - 12x - 24$$

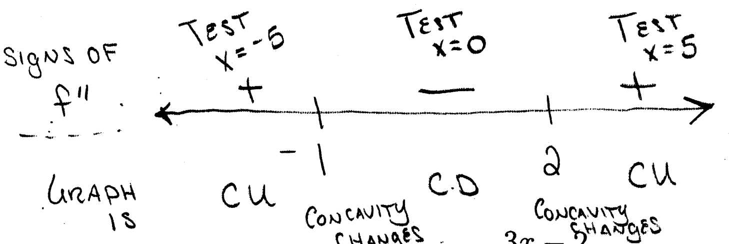
$$= 12(x^2 - x - 2) = 12(x-2)(x+1)$$

$$f''(x) = 0 \Rightarrow X = 2,\ X = -1$$

GRAPH IS CD ON $(-1,2)$

GRAPH IS CU ON $(-\infty,-1) \cup (2,\infty)$

$(-1, f(-1)) = (-1,-43)$ & $(2,f(2)) = (2,26)$ ARE INFLECTION PTS



10. (6 points) The graph of $y = \dfrac{3x-2}{\sqrt{4x^2+5}}$ has two horizontal asymptotes. Find either one of them. Show all work.

$$\lim_{X\to\infty} \frac{3X-2}{\sqrt{4X^2+5}} \cdot \frac{\frac{1}{X}}{\frac{1}{\sqrt{X^2}}} = \lim_{X\to\infty} \frac{3-\frac{2}{X}}{\sqrt{4+\frac{5}{X^2}}} = \frac{3-0}{\sqrt{4+0}} = \frac{3}{2}$$

$$\boxed{y = \frac{3}{2}}$$

THE OTHER H.A. IS $y = -\frac{3}{2}$.

11. (10 points) Tell whether each statement is true or false.

(a) FALSE L'Hôpital's rule can be used to evaluate a limit involving any kind of indeterminate form. $\frac{0}{0}$ OR $\frac{\infty}{\infty}$

(b) TRUE If $f'(5) = 0$ and $f''(5) = 10$, then $f(5)$ is a relative minimum.

(c) FALSE Suppose that f is a function for which $f''(x) = x^4$. The graph of f has an inflection point at $x = 0$.

SIGNS OF f'' $\quad + \quad | \quad +$

$\quad\quad\quad\quad\quad\quad 0$

(d) FALSE Every absolute extreme value is also a relative extreme value.

(e) FALSE If $y = \sin x$, then $dy = \cos x$.

$dy = \cos x \, dx$

12. (6 points) Find the critical numbers of $f(x) = \dfrac{4x^2 - 11x + 9}{x}$. Also, explain why $x = 0$ is not a critical number.

$X = 0$ IS NOT IN THE DOMAIN OF f.

$$f'(x) = \frac{x(8x-11) - (4x^2-11x+9)(1)}{x^2} = \frac{4x^2-9}{x^2}$$

$$f'(x) = 0 \Rightarrow 4x^2 - 9 = 0$$

$$(2x+3)(2x-3) = 0 \qquad \boxed{x = \pm \frac{3}{2}}$$

13. (4 points) Tell why L'Hôpital's rule does not apply to each limit.

(a) $\displaystyle\lim_{x\to 2} \frac{x^2+3x}{x^2+9} = \frac{10}{13}$ NOT A $\frac{0}{0}$ OR $\frac{\infty}{\infty}$ FORM!

(b) $\displaystyle\lim_{x\to\infty} xe^{-x}$ THIS LIMIT HAS THE FORM $\infty \cdot 0$.

IT IS INDETERMINATE, BUT L'HÔPITAL ONLY APPLIES TO $\frac{0}{0}$ OR $\frac{\infty}{\infty}$.

Math 131 - Test 3 (TH)

April 17, 2023

Name key

Score

Show all work to receive full credit. Supply explanations where necessary. This problem is due April 24.

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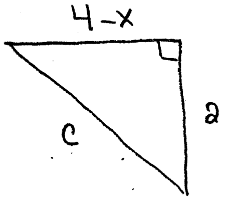
1. (10 points) In this problem, you will use calculus techniques to optimize a function in an application. **The Problem:** Starting a point A, a company must lay cable to point B. It is 2 times more expensive to lay the cable through the field than along the road. Referring the the figure, you will find the x-value that minimizes the overall cost.

   (a) The length of the cable through the field is $\sqrt{4+(4-x)^2}$. Explain where this expression comes from. Also expand the polynomial under the radical and combine like terms.

PYTHAG THEOREM



$$c^2 = (4-x)^2 + 2^2 = 20-8x+x^2$$

$$c = \sqrt{20-8x+x^2}$$

   (b) The cost of laying the cable along the road is k dollars per mile. Therefore, the cost through the field is $2k$ dollars per mile. This makes the overall cost of the project

$$C(x) = kx + 2k\sqrt{20-8x+x^2}, \text{ where } 0 \le x \le 4.$$

   Determine $C'(x)$.

$$C'(x) = k + k\left(20-8x+x^2\right)^{-1/2}(2x-8) = k + \frac{(2x-8)k}{\sqrt{20-8x+x^2}}$$

   (c) Determine the critical number of C. The algebra will be a little bit messy. Feel free to use technology to solve the necessary equation. (The critical number does not depend on k–you can just ignore it.)

$$k + \frac{(2x-8)k}{\sqrt{20-8x+x^2}} = 0 \Rightarrow 2x-8 = -\sqrt{20-8x+x^2}$$

$$4x^2-32x+64 = 20-8x+x^2$$

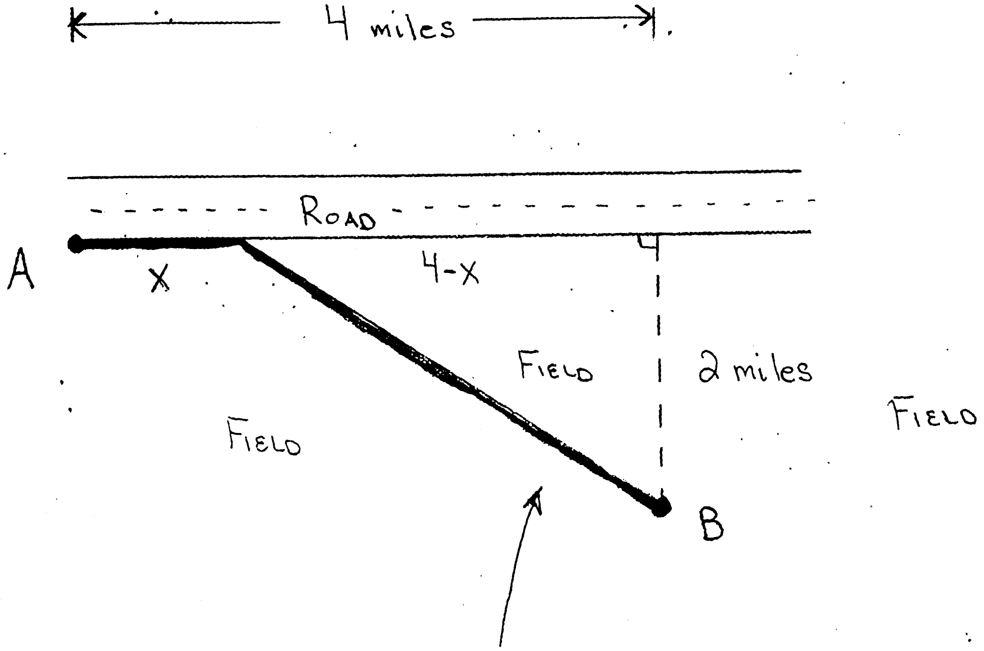
$$3x^2-24x+44=$$

$$x = 4 - \frac{2\sqrt{3}}{3} \approx 2.8453$$

(THE OTHER SOLUTION IS GREATER THAN 4.)

   (d) Show that your critical number gives an absolute minimum.

x	C(x)	
0	8.944272 k	← ABS MAX
4.	8k	
2.8453	7.464102 k	← ABS MIN.



CABLE IS LAID IN TWO SEGMENTS: ONE ALONG THE ROAD AND ONE THROUGH THE FIELD.