Math 131 - Test 3

Name _____

April 17, 2023

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) Use logarithmic differentiation to find dy/dx when $x = 3$.

$$y = \frac{x^2(x-2)^5}{x^2+16}$$

2. (4 points) Evaluate the limit: $\lim_{x\to-\infty}\left(\frac{x^4-4x^3+1}{2-2x^2-7x^4}\right)$

3. (8 points) Determine each derivative.

(a) $\dfrac{d}{dx}[x \sin^{-1}(x^2)]$

(b) $\dfrac{d}{dx} 2e^{\tan x}$

4. (6 points) Some values of $f(x)$ and $f'(x)$ are given in the table below.

x	-1	0	1	2
$f(x)$	-0.5737	0.7027	1.1044	11.2234
$f'(x)$	0.0740	0.2082	0.7423	171.4132

(a) Determine the linearization of f at $x = 1$.

(b) Use the linearization you found above to approximate $f(0.925)$.

5. (6 points) Let $y = \dfrac{1}{2x+1}$. Use differentials to approximate Δy as x changes from $x = 1$ to $x = 1.25$.

6. (8 points) Use calculus techniques to determine the absolute minimum and maximum values of $f(x) = 5x^2 - 6x^{5/3}$ over $[0, 2]$.

7. (8 points) Evaluate the limit: $\displaystyle\lim_{x\to 0}\left(\frac{e^x - 4x^2 - 2 + e^{-x}}{x^2}\right)$

8. (8 points) The function $f(x) = 5x^{1/3} - x^{5/3}$ has exactly three critical numbers: $x = -1$, $x = 0$, and $x = 1$. Use calculus techniques to identify all relative extreme values of f.

9. (8 points) Let $f(x) = x^4 - 2x^3 - 12x^2 + 36x + 2$. Find open intervals on which the graph of f is concave up/down. Also identify all inflection points.

10. (6 points) The graph of $y = \dfrac{3x - 2}{\sqrt{4x^2 + 5}}$ has two horizontal asymptotes. Find either one of them. Show all work.

11. (10 points) Tell whether each statement is true or false.

(a) _____ L'Hôpital's rule can be used to evaluate a limit involving any kind of indeterminate form.

(b) _____ If $f'(5) = 0$ and $f''(5) = 10$, then $f(5)$ is a relative minimum.

(c) _____ Suppose that f is a function for which $f''(x) = x^4$. The graph of f has an inflection point at $x = 0$.

(d) _____ Every absolute extreme value is also a relative extreme value.

(e) _____ If $y = \sin x$, then $dy = \cos x$.

12. (6 points) Find the critical numbers of $f(x) = \dfrac{4x^2 - 11x + 9}{x}$. Also, explain why $x = 0$ is not a critical number.

13. (4 points) Tell why L'Hôpital's rule does not apply to each limit.

(a) $\lim\limits_{x \to 2} \dfrac{x^2 + 3x}{x^2 + 9}$

(b) $\lim\limits_{x \to \infty} xe^{-x}$

Math 131 - Test 3 (TH)

Name _____

April 17, 2023

Score _____

Show all work to receive full credit. Supply explanations where necessary. This problem is due April 24.

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1. (10 points) In this problem, you will use calculus techniques to optimize a function in an application. **The Problem:** Starting a point A, a company must lay cable to point B. It is 2 times more expensive to lay the cable through the field than along the road. Referring the the figure, you will find the x-value that minimizes the overall cost.

   (a) The length of the cable through the field is $\sqrt{4+(4-x)^2}$. Explain where this expression comes from. Also expand the polynomial under the radical and combine like terms.

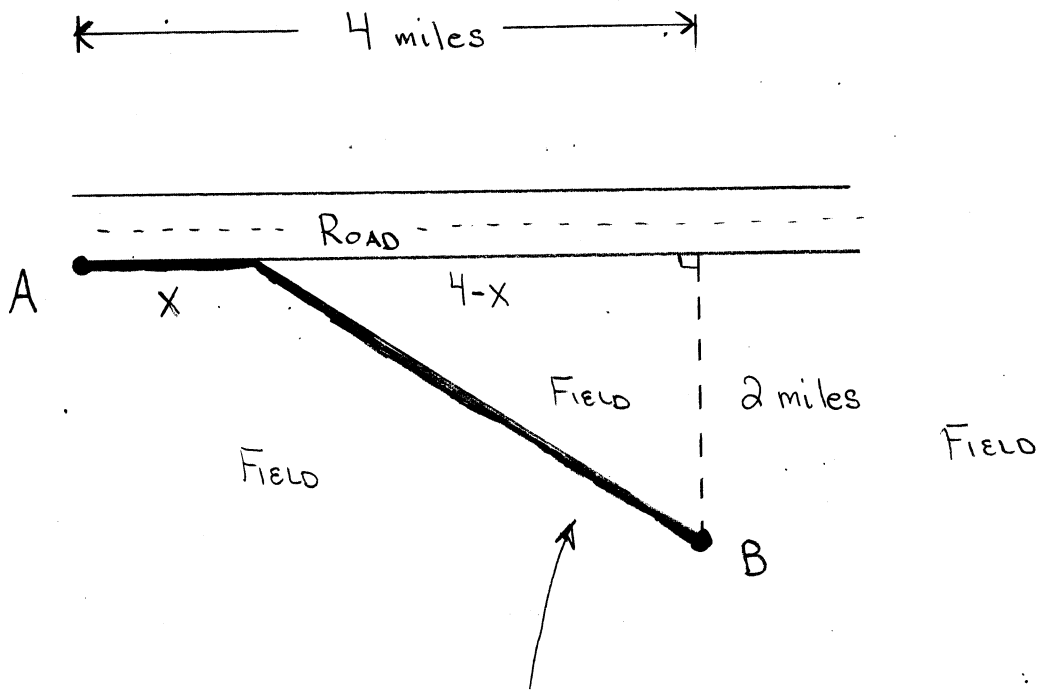
   (b) The cost of laying the cable along the road is k dollars per mile. Therefore, the cost through the field is $2k$ dollars per mile. This makes the overall cost of the project

   $$C(x) = kx + 2k\sqrt{20-8x+x^2}, \text{ where } 0 \le x \le 4.$$

   Determine $C'(x)$.

   (c) Determine the critical number of C. The algebra will be a little bit messy. Feel free to use technology to solve the necessary equation. (The critical number does not depend on k–you can just ignore it.)

   (d) Show that your critical number gives an absolute minimum.



CABLE IS LAID IN TWO SEGMENTS: ONE ALONG THE ROAD AND ONE THROUGH THE FIELD.