Math 131 - Final Exam

Name _____

May 8, 2023

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Use algebraic techniques (not a graph, table, or L'Hôpital's rule) to determine each limit.

   (a) $\lim_{r \to 1} \dfrac{\sqrt{r} - 1}{r - 1}$

   (b) $\lim_{x \to 6} \dfrac{(x+3)^2 - 5(x+3) - x^2}{3(x-6)}$

2. (10 points) Use the definition of continuity to explain why f is discontinuous at $x = 5$. Also state the type of discontinuity.

$$f(x) = \begin{cases} 4x + 5, & x < 5 \\ x^2 + x\cos(\pi x), & x \geq 5 \end{cases}$$

3. (10 points) Let $f(x) = x^2 - 4x$. Use the **limit definition of the derivative** to determine $f'(x)$. Show all work.

4. (10 points) Use basic differentiation rules to determine each derivative. Do not simplify.

   (a) $\dfrac{d}{dx}\left(\dfrac{\tan^{-1} x}{1 + 2x + x^2}\right)$

   (b) $\dfrac{d}{dx}\left[e^{-5x^2} \cot x\right]$

5. (10 points) Let $f(x) = \sin(x) + \ln(x^2)$. Find the linearization of f at $x = 1$. Then use your linearization to approximate $f(0.8)$. Write your answers with all numbers in decimal form, rounded to three places. (Make sure your calculator is in radian mode.)

6. (10 points) Use any analytical method (not a table or graph) to determine each limit.

(a) $\lim_{x \to -\infty} \left(\dfrac{3x^4 - 5x^2 + 8}{3x^2 + 4x^4 + 5x^4} \right)$

(b) $\lim_{x \to 0} \left(\dfrac{x - \sin 5x}{x^2} \right)$

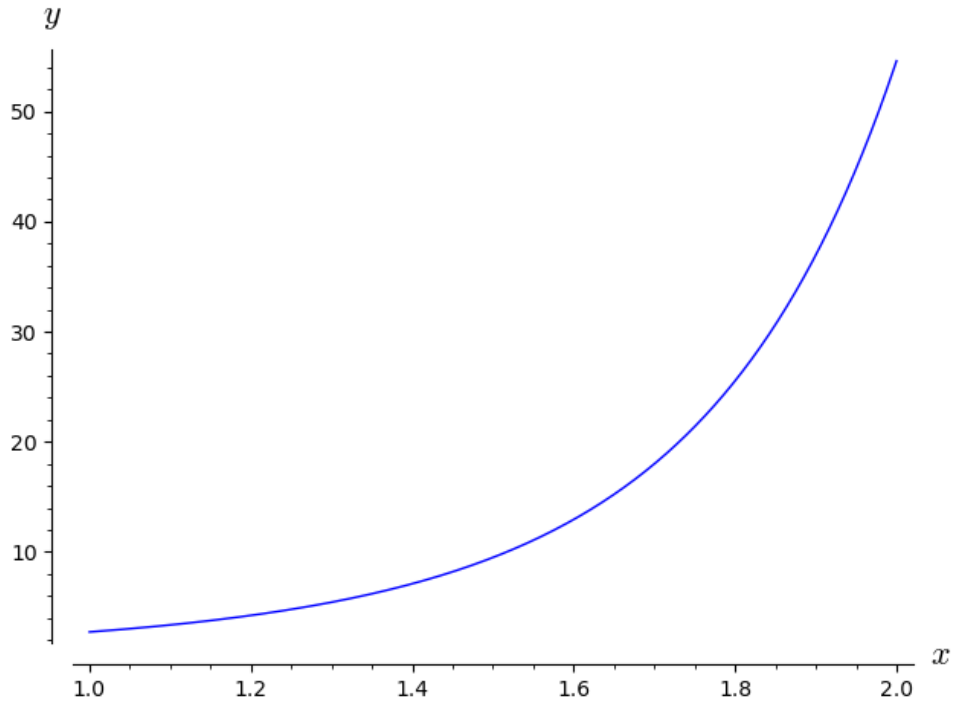
7. (10 points) Use calculus techniques to find the **absolute extreme values** of $g(x) = x^4 + 4x^3 - 20x^2$ on the interval $[-2, 3]$.

8. (10 points) Evaluate each definite integral. (You may need to use a substitution.)

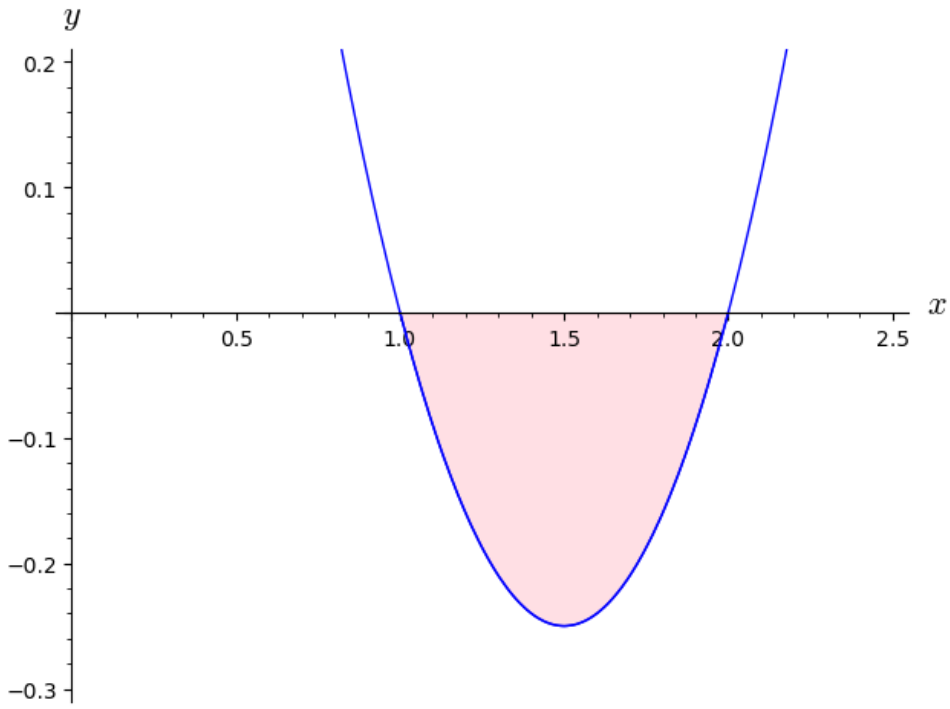
   (a) $\displaystyle\int_1^2 \frac{1 + t + t^2}{t^3}\, dt$

   (b) $\displaystyle\int_0^{\pi} (1 + \cos x)^3 \sin x\, dx$

9. (10 points) The graph of $f(x) = e^{x^2}$ over the interval $[1, 2]$ is shown below. Use five subintervals of equal length and subinterval **midpoints** to compute the corresponding (middle) Riemann sum for f on $[1, 2]$. Once you have computed the Riemann sum, sketch the corresponding rectangles on the graph.



10. (10 points) The graph of $y = x^2 - 3x + 2$ is shown below.



(a) By solving $y = 0$ (Show your work!), show algebraically that the x-intercepts of the graph agree with those shown.

(b) Use the fundamental theorem of calculus to find the area of the shaded region.