Math 131 - Assignment 11

April 24, 2024

Name Key

Score ________

Show all work to receive full credit. Supply explanations when necessary. Use extra paper as necessary. This assignment is due May 1.

1. Find the function f that satisfies $f'(x) = 9x^2 - 3x + 4\sin x$ and $f(0) = 7$.

$$\int (9x^2 - 3x + 4\sin x)\,dx = 3x^3 - \frac{3}{2}x^2 - 4\cos x + C$$

$$f(0) = 7 \Rightarrow -4 + C = 7 \Rightarrow C = 11$$

$$\boxed{f(x) = 3x^3 - \frac{3}{2}x^2 - 4\cos x + 11}$$

2. Let $f(x) = \sin(x)$. Use 4 subintervals of equal length and right endpoints to compute the corresponding right Riemann sum for f over the interval $[1, 2]$.

$\Delta x = \frac{2-1}{4}$

$x_0 = 1$

$x_1 = 1.25$ $\quad \Delta x = 0.25$

$x_2 = 1.5$

$x_3 = 1.75$

$x_4 = 2$

Right endpts:

$c_1 = 1.25$

$c_2 = 1.5$

$c_3 = 1.75$

$c_4 = 2$

Riemann sum $=$

$\sin(1.25)(0.25) + \sin(1.5)(0.25) + \sin(1.75)(0.25) + \sin(2)(0.25)$

≈ 0.9599

3. Let $f(x) = \frac{1}{x}$. Use 6 subintervals of equal length and subinterval left endpoints to compute the corresponding Riemann sum for f over the interval $[1, 4]$.

$\Delta x = \frac{4-1}{6} = \frac{1}{2}$

$x_0 = 1$

$x_1 = 1.5$ $\quad \Delta x = \frac{1}{2}$

$x_2 = 2$

$x_3 = 2.5$

$x_4 = 3$

$x_5 = 3.5$

$x_6 = 4$

$c_1 = 1$

$c_2 = 1.5$

$c_3 = 2$

$c_4 = 2.5$

$c_5 = 3$

$c_6 = 3.5$

Riemann sum $=$

$\frac{1}{1}(0.5) + \frac{1}{1.5}(0.5) + \frac{1}{2}(0.5) + \frac{1}{2.5}(0.5) + \frac{1}{3}(0.5) + \frac{1}{3.5}(0.5)$

≈ 1.5929

4. Use 4 subintervals of equal length and subinterval midpoints to compute a Riemann sum for $f(x) = \sin(x^2)$ on the interval $[0, 1]$.

$\Delta x = 0.25$

$x_0 = 0$
$x_1 = 0.25$
$x_2 = 0.5$
$x_3 = 0.75$
$x_4 = 1$

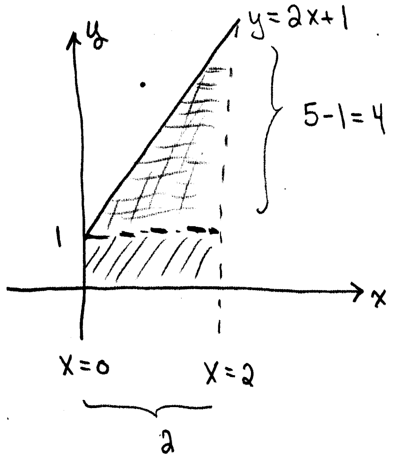
$c_1 = 0.125$
$c_2 = 0.375$
$c_3 = 0.625$
$c_4 = 0.875$

Riemann sum =

$$\sin(0.125^2)(0.25) + \sin(0.375^2)(0.25) + \sin(0.625^2)(0.25) + \sin(0.875^2)(0.25)$$

$$\approx 0.3074$$

5. Use the area concept (not a Riemann sum or antidifferentiation) to evaluate $\int_0^2 (2x+1)\,dx$. Show your work.

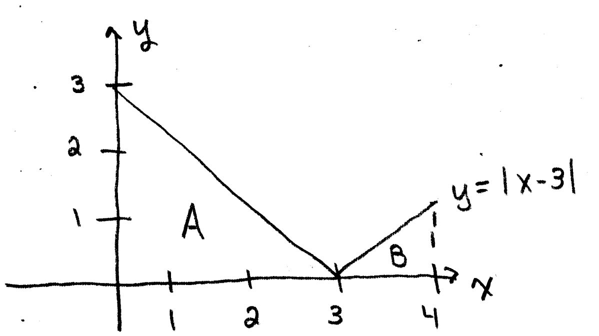


$$\int_0^2 (2x+1)\,dx = \text{Area of Rect} + \text{Area of Tri}$$

$$= 2(1) + \tfrac{1}{2}(4)(2)$$

$$= 2 + 4 = \boxed{6}$$

6. Sketch the graph of $y = |x - 3|$ over the interval from $x = 0$ to $x = 4$. Then use area (not a Riemann sum or antidifferentiation) to determine the value of the definite integral $\int_0^4 |x - 3|\,dx$.



$$\int_0^3 |x-3|\,dx = A + B$$

$$= \tfrac{1}{2}(3)(3) + \tfrac{1}{2}(1)(1)$$

$$= \tfrac{9}{2} + \tfrac{1}{2} = \boxed{5}$$

7. Use the fundamental theorem of calculus to evaluate each definite integral.

(a) $\displaystyle\int_0^{\pi/2} (x + \sin x)\, dx = \frac{1}{2}x^2 - \cos x \Big|_0^{\pi/2} = \left(\frac{\pi^2}{8} - 0\right) - \left(0 - 1\right)$

$= \boxed{1 + \frac{\pi^2}{8}}$

(b) $\displaystyle\int_1^2 \frac{1+x}{x}\, dx = \int_1^2 \left(\frac{1}{x} + 1\right) dx = \ln|x| + x \Big|_1^2$

$= (\ln(2) + 2) - (\ln(1) + 1)$

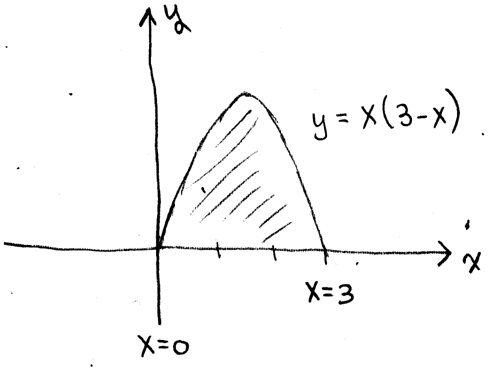
$= \boxed{\ln(2) + 1}$

(c) $\displaystyle\int_0^{\pi} \cos x\, dx = \sin x \Big|_0^{\pi} = \sin \pi - \sin 0 = \boxed{0}$

(d) $\displaystyle\int_1^2 \left(\frac{1}{x} - e^x\right) dx = \ln|x| - e^x \Big|_1^2 = (\ln(2) - e^2) - (\ln(1) - e)$

$= \boxed{\ln(2) - e^2 + e}$

8. Use a definite integral to find the area of the bounded region above the x-axis and below the graph of $y = 3x - x^2 = x(3-x)$



$\displaystyle\int_0^3 (3x - x^2)\, dx = \frac{3}{2}x^2 - \frac{1}{3}x^3 \Big|_0^3$

$= \left(\frac{27}{2} - \frac{27}{3}\right) - (0 - 0)$

$= \boxed{\frac{9}{2}}$