Math 131 - Assignment 2

January 24, 2024

Name Key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due January 31.

1. Find the limit analytically: $\lim_{x\to -5}\left(\frac{x^2+3x-10}{x^3+11x^2+30x}\right)$

0/0 MORE WORK!

$$= \lim_{x\to -5} \frac{(x+5)(x-2)}{x(x+5)(x+6)} = \frac{-7}{-5} = \boxed{\frac{7}{5}}$$

2. Find the limit analytically: $\lim_{x\to 1}\frac{\sqrt{x+3}-2}{x-1}$

0/0 MORE WORK

$$\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1}\cdot\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2} = \lim_{x\to 1}\frac{x+3-4}{(x-1)(\sqrt{x+3}+2)} = \lim_{x\to 1}\frac{x-1}{(x-1)(\sqrt{x+3}+2)}$$

$$= \boxed{\frac{1}{4}}$$

3. Find the limit analytically: $\lim_{w\to 0}\frac{(w+6)^2-36}{w}$

0/0 MORE WORK

$$\lim_{w\to 0}\frac{w^2+12w+36-36}{w} = \lim_{w\to 0}\frac{w^2+12w}{w} = \lim_{w\to 0}(w+12) = \boxed{12}$$

0/0 MORE WORK.

4. Find the limit analytically: $\lim_{x\to 4}\frac{\frac{1}{2}-\frac{1}{x-2}}{x-4}$

$$\lim_{x\to 4}\frac{\frac{1}{2}\frac{(x-2)}{(x-2)}-\frac{1}{x-2}\frac{(2)}{(2)}}{x-4} = \lim_{x\to 4}\frac{\frac{x-2-2}{2(x-2)}}{x-4} = \lim_{x\to 4}\frac{x-4}{2(x-2)(x-4)} = \frac{1}{2(4-2)}$$

$$= \boxed{\frac{1}{4}}$$

5. Explain why direct substitution cannot be used to evaluate the limit: $\lim_{x\to 1}\sqrt{1-x^2}$

$\sqrt{1-x^2}$ IS DEFINED ONLY FOR $-1 \le x \le 1$.

Turn over.

THE LIMIT CANNOT EXIST BECAUSE THE FUNCTION IS NOT DEFINED TO THE RIGHT OF $x=1$, EVEN THOUGH DIRECT SUBS GIVES 0.

0/0

6. Find the limit analytically: $\lim_{y\to 0} \dfrac{\tan(6y)}{3y}$

REWRITE $\dfrac{\tan 6y}{3y} = \dfrac{1}{3y}\cdot\dfrac{\sin 6y}{\cos 6y}\cdot\dfrac{6}{6}$

$= \dfrac{6}{3\cos 6y}\cdot\dfrac{\sin 6y}{6y}$

$= \dfrac{2}{\cos 6y}\cdot\dfrac{\sin 6y}{6y}$

$\lim_{y\to 0} \dfrac{\tan 6y}{3y} = \lim_{y\to 0} \dfrac{2}{\cos 6y}\cdot\dfrac{\sin 6y}{6y}$

$= \left(\dfrac{2}{1}\right)(1) = \boxed{2}$

7. Find the limit analytically: $\lim_{t\to 3^-} \dfrac{t^2-t-6}{|t-3|}$

0/0 MORE WORK

$\lim_{t\to 3^-} \dfrac{(t-3)(t+2)}{|t-3|} = \lim_{t\to 3^-} \dfrac{(t-3)(t+2)}{-(t-3)} = \dfrac{5}{-1} = \boxed{-5}$

$|t-3| = -(t-3)$ FOR $t<3$

8. Determine the value of the constant k so that $\lim_{x\to 4} g(x)$ exists.

$$g(x) = \begin{cases} kx+\sin(\pi x), & x \le 4 \\ x\cos(\pi x) - x^2, & x > 4 \end{cases}$$

$\lim_{x\to 4^-} g(x) = \lim_{x\to 4} (kx + \sin(\pi x))$

$= 4k + \sin(4\pi) = 4k$

$\lim_{x\to 4^+} g(x) = \lim_{x\to 4} [x\cos(\pi x) - x^2]$

$= 4\cos(4\pi) - 16 = -12$

$4k = -12 \Rightarrow \boxed{k = -3}$

9. Find the limit analytically: $\lim_{x\to -2^+} (5x^2 - 10x + 13)$

$= 5(-2)^2 - 10(-2) + 13$

$= 20 + 20 + 13 = \boxed{53}$

10. Give an example of a one-sided limit that does not exist and say why.

$\lim_{x\to 1^+} \sqrt{1-x^2}$ DNE. SEE PROBLEM #5 ABOVE.