Math 131 - Assignment 3

January 31, 2024

Name key

Score

Show all work to receive full credit. Evaluate all limits analytically. Supply explanations when necessary. This assignment is due February7.

1. In each part below, determine analytically (not with a table or graph) whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

   (a) $\displaystyle\lim_{x\to 2^-} \frac{x}{x-2}$ $2/0$ UNBOUNDED GROWTH

   LEFT OF $X=2$: $\dfrac{X}{X-2} = \dfrac{+}{-} \Rightarrow$ $\boxed{\text{LIMIT IS } -\infty}$

   (b) $\displaystyle\lim_{x\to 2^+} \frac{x}{x-2}$ $2/0$ UNBOUNDED GROWTH

   RIGHT OF $X=2$: $\dfrac{X}{X-2} = \dfrac{+}{+} \Rightarrow$ $\boxed{\text{LIMIT IS } +\infty}$

   (c) $\displaystyle\lim_{x\to 2} \frac{x}{x-2}$

   $\boxed{\text{LIMIT DNE}}$ FROM PARTS (a) & (b)

   (d) $\displaystyle\lim_{x\to 2} \frac{x}{(x-2)^4}$ $2/0$ UNBOUNDED GROWTH

   BOTH SIDES OF $X=2$: $\dfrac{X}{(X-2)^4} = \dfrac{+}{+} \Rightarrow$ $\boxed{\text{LIMIT IS } +\infty}$

2. Evaluate $\displaystyle\lim_{x\to -3^+} h(x)$, where $h(x) = \begin{cases} \tan(\pi x/2), & x < -3 \\ 2x^2 + x\cos(\pi x), & x > -3 \end{cases}$

   $$\lim_{X\to -3} \left(2x^2 + x\cos(\pi x)\right) = 2(-3)^2 - 3\cos(-3\pi)$$
   $$= 18+3 = \boxed{21}$$

3. Evaluate the limit: $\displaystyle\lim_{w\to 4^-} \frac{2w^2-8w}{w^2-8w+16}$ $0/0$ MORE WORK

   $$\lim_{w\to 4^-} \frac{2w(w-4)}{(w-4)^2} = \lim_{w\to 4^-} \frac{2w}{w-4}$$ $8/0$ UNBOUNDED GROWTH

   LEFT OF $w=4$: $\dfrac{2w}{w-4} = \dfrac{+}{-} \Rightarrow$ $\boxed{\text{LIMIT IS } -\infty}$

4. Evaluate the limit: $\displaystyle\lim_{x\to 0^+} \frac{\sin 2x}{\sin 5x}$. $0/0$ MORE WORK

   REWRITE $\dfrac{\sin 2x}{\sin 5x}$

   LIKE THIS

   $$\frac{2}{5}\cdot\frac{\sin 2x}{2x}\cdot\frac{5x}{\sin 5x}$$

   $$\lim_{X\to 0^+} \frac{2}{5}\cdot\frac{\sin 2x}{2x}\cdot\frac{5x}{\sin 5x}$$
   $$= \left(\frac{2}{5}\right)\cdot(1)\cdot(1) = \boxed{\frac{2}{5}}$$

Turn over.

5. Determine all vertical asymptotes of the graph of $h(x) = \dfrac{x^2+2x-8}{x^2-4}$.
(You can use your graphing calculator for help, but you must show computational work for credit.)

$$h(x) = \frac{(x+4)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$$

X = 2 IS A REMOVABLE DISCONT.

X = −2 IS ASSOCIATED WITH NONZERO/ZERO.

$$\boxed{\text{V. A. } x = -2}$$

6. Find the numbers a and b so that f is continuous everywhere.

$$f(x) = \begin{cases} x^2+ax+b, & x<3 \\ bx+a, & x=3 \\ x^3-9x+bx^2, & x>3 \end{cases}$$

MUST HAVE

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$$

$f(3) = 3b+a$

$(3)^2 + 3a + b = 9 + 3a + b$

$(3)^3 - 9(3) + b(3)^2 = 9b$

$9 + 3a + b = 3b + a$

$9b = 3b + a$

$2a - 2b = -9$

$-a + 6b = 0 \Rightarrow a = 6b$

$10b = -9$

$b = \dfrac{-9}{10}$

$a = \dfrac{-54}{10}$

7. Find and classify the discontinuities of $R(x) = \dfrac{2x^2-5x-3}{x^2+4x-21}$.

$$R(x) = \frac{(2x+1)\cancel{(x-3)}}{(x+7)\cancel{(x-3)}}$$

X = 3 IS A REMOVABLE DISCONT

X = −7 IS AN INFINITE DISCONT

8. Suppose that the function f satisfies

$$1 - x \le f(x) \le 1 - x + \frac{x^2}{2}$$

for all x-values. Determine the limit, $\lim_{x\to 0} f(x)$, and explain your reasoning.

$$\lim_{x\to 0} (1-x) = 1$$

$$\lim_{x\to 0} \left(1 - x + \frac{x^2}{2}\right) = 1$$

$$\Rightarrow \lim_{x\to 0} f(x) = 1$$

BY SQUEEZE THM.