Math 131 - Assignment 4

February 14, 2024

Name key

Score

Show all work to receive full credit. Evaluate all limits analytically. Supply explanations when necessary. This assignment is due February 21.

1. Let $f(x) = 2 - x + \sin x$. Find an interval of length one that contains a solution of the equation $f(x) = 0$. Then use the Intermediate Value Theorem to explain how you know.

f IS CONTINUOUS EVERYWHERE AND $f(2) \approx 0.909$ AND $f(3) \approx -0.859$.

ACCORDING TO THE IVT, f MUST TAKE ON EACH y-VALUE BETWEEN -0.859 & 0.909 ON $[2,3]$. IN PARTICULAR, $f(x) = 0$ FOR SOME X IN $[2,3]$.

2. Use the limit definition of derivative to determine $f'(x)$ when $f(x) = 3x^2 - x + 1$. (Use extra paper as necessary.)

$$\lim_{h\to 0} \frac{[3(x+h)^2 - (x+h) + 1] - [3x^2 - x + 1]}{h} = \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 - x - h + 1 - 3x^2 + x - 1}{h}$$

$$= \lim_{h\to 0} \frac{6xh + 3h^2 - h}{h} = \lim_{h\to 0} (6x + 3h - 1) = 6x - 1$$

3. Let $f(x) = \sqrt{x^2 + 16}$. We will soon learn that $f'(x) = \dfrac{x}{\sqrt{x^2+16}}$. For now, just use the given information to find an equation of the line tangent to the graph of f at the point where $x = 3$.

POINT: $X = 3$, $y = f(3) = \sqrt{25} = 5$

$(3,5)$

SLOPE: $f'(3) = \dfrac{3}{\sqrt{25}} = \dfrac{3}{5}$

TAN LINE:

$y - 5 = \frac{3}{5}(x-3)$

OR

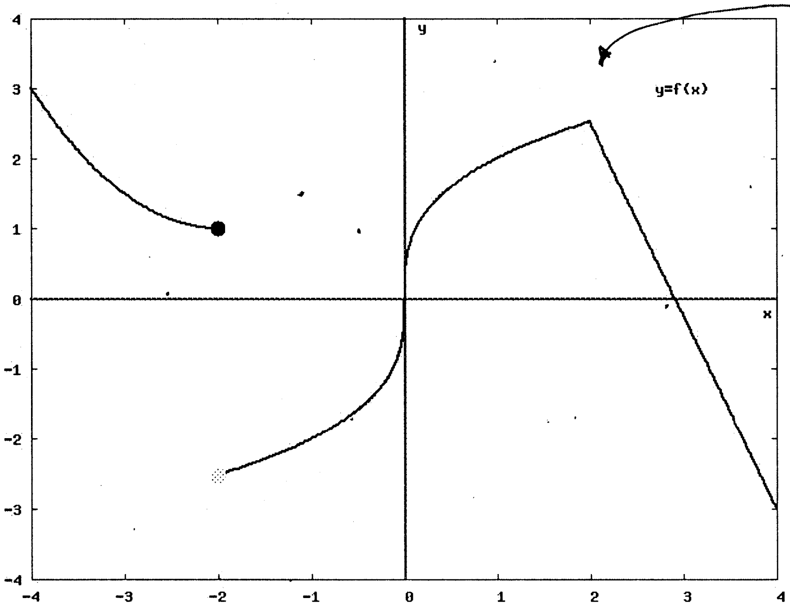
$y = \frac{3}{5}x + \frac{16}{5}$

4. Use the limit definition of derivative to determine $f'(x)$ when $f(x) = \dfrac{1}{x}$. (Use extra paper as necessary.)

$$\lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \lim_{h\to 0} \frac{\frac{-h}{x(x+h)}}{h} = \lim_{h\to 0} \frac{-h}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}$$

5. The graph of $y = f(x)$ is shown below. Give the x-coordinates of three points at which $f'(x)$ does not exist. For each point, very briefly say why f' does not exist.



$f'(2)$ DNE BECAUSE graph HAS SHARP POINT: TAN LINE FROM RIGHT IS NOT THE TAN LINE FROM LEFT.

$f'(-2)$ DNE BECAUSE f IS NOT CONTINUOUS.

$f'(0)$ DNE BECAUSE TANGENT LINE IS VERTICAL

6. Use basic derivative rules (not the definition) to find each derivative.

(a) $\dfrac{d}{dx}\left(7x^3 - 10 + \sqrt[3]{x} - \dfrac{7}{x^3}\right) = \dfrac{d}{dx}\left(7x^3 - 10 + x^{1/3} - 7x^{-3}\right)$

$$= \boxed{21x^2 + \frac{1}{3}x^{-2/3} + 21x^{-4} = 21x^2 + \frac{1}{3\sqrt[3]{x^2}} + \frac{21}{x^4}}$$

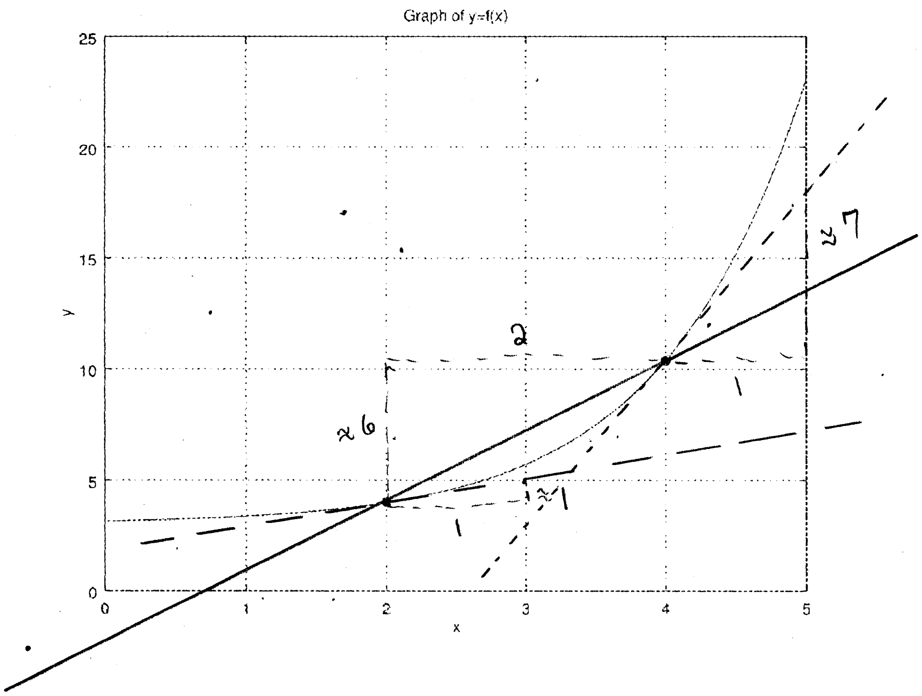
(b) $\dfrac{d}{dx}(5\sin x - 12\cos x)$

$$= \boxed{5\cos x + 12\sin x}$$

(c) $\dfrac{d}{dx}\dfrac{5x^4 + 8x - 3}{x^2}$ (Do not use the quotient rule. Use algebra to rewrite the expression before you differentiate.)

$$= \frac{d}{dx}\left(5x^2 + 8x^{-1} - 3x^{-2}\right) = \boxed{10x - 8x^{-2} + 6x^{-3} = 10x - \frac{8}{x^2} + \frac{6}{x^3}}$$

7. The graph of $y = f(x)$ is shown below.



(a) Sketch the secant line through the indicated points at $x = 2$ and $x = 4$. Let m be the slope of the secant line through those points. Estimate the value of m.

Roughly, $\frac{\Delta y}{\Delta x} = \frac{6}{2} = \boxed{3}$

(b) Sketch the tangent line through the point where $x = 2$. Use your tangent line to estimate $f'(2)$.

Roughly looks like $\frac{1}{1} = \boxed{1}$

(c) Sketch the tangent line through the point where $x = 4$. Use your tangent line to estimate $f'(4)$.

Roughly $\frac{7}{1} \approx \boxed{7}$

8. Find an equation of the line tangent to the graph of $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ at the point where $x = 4$.

$\leftarrow x^{1/2} + x^{-1/2}$

Point: $x = 4,\ y = 2 + \frac{1}{2} = \frac{5}{2}$

Slope: $\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$

$m = \left.\frac{dy}{dx}\right|_{x=4} = \frac{1}{4} - \frac{1}{16} = \frac{3}{16}$

Tan line:

$y - \frac{5}{2} = \frac{3}{16}(x-4)$

or

$y = \frac{3}{16}x + \frac{7}{4}$