Math 131 - Assignment 5

February 21, 2024

Name Key

Score

Show all work to receive full credit. Supply explanations when necessary. Use extra paper as necessary. This assignment is due February 28.

1. Use the quotient rule and trig identities to derive the formula $\frac{d}{dx}\sec x = \sec x \tan x$.

$$\frac{d}{dx}\frac{1}{\cos x} = \frac{(\cos x)(0) - (1)(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x}\frac{\sin x}{\cos x}$$

$$= \sec x \tan x \quad ✓$$

2. Let $g(x) = x^2\left(\frac{2}{x} - \frac{1}{x+1}\right)$. Find $g'(x)$. Then find the slope of the line tangent to the graph of g at the point where $x = 1$.

$$g(x) = 2x - \frac{x^2}{x+1} \qquad g'(x) = 2 - \frac{(x+1)(2x) - x^2(1)}{(x+1)^2} = \boxed{2 - \frac{x^2+2x}{(x+1)^2}}$$

$$g'(1) = 2 - \frac{3}{4} = \boxed{\frac{5}{4}}$$

3. Find $\frac{dy}{dx}$ when $y = \frac{3(1-\sin x)}{2\cos x}$.

$$\boxed{\frac{dy}{dx} = \frac{(2\cos x)(-3\cos x) - (3)(1-\sin x)(-2\sin x)}{4\cos^2 x}} = \frac{-6\cos^2 - 6\sin^2 x + 6\sin x}{4\cos^2 x}$$

$$= \frac{-6+6\sin x}{4\cos^2 x}$$

4. Let $f(x) = \frac{x^2}{x^2+1}$. Find $f'(x)$. Then find all points at which the graph of f has a horizontal tangent line.

$$\boxed{f'(x) = \frac{(x^2+1)(2x) - (x^2)(2x)}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}}$$

$$f'(x) = 0$$

$$\Rightarrow 2x = 0 \Rightarrow \boxed{x = 0}$$

5. Let $g(x) = 5x^3 + 8x^2 - 18x + 17$. Find $g'(x)$, $g''(x)$, $g'''(x)$, and $g^{(4)}(x)$.

$$g'(x) = 15x^2 + 16x - 18$$

$$g''(x) = 30x + 16$$

$$g'''(x) = 30$$

$$g^{(4)}(x) = 0$$

6. A ball is dropped from a height of 100 feet. Its height s (in feet) after t seconds is given by

$$s(t) = -16t^2 + 100.$$

(a) Find the average velocity from $t = 1$ to $t = 1.5$.

$$\frac{s(1.5) - s(1)}{1.5 - 1} = \frac{64 - 84}{0.5} = \frac{-20}{0.5} = \boxed{-40 \text{ FT/SEC}}$$

(b) Find the instantaneous velocity at $t = 1.5$.

$$s'(t) = -32t \qquad \boxed{s'(1.5) = -48 \text{ FT/SEC}}$$

(c) Find the instantaneous velocity when the ball hits the ground.

$$s(t) = 0 \Rightarrow -16t^2 + 100 = 0$$

$$t = \sqrt{\frac{100}{16}} = 2.5 \qquad \boxed{s'(2.5) = -80 \text{ FT/SEC}}$$

7. An object is launched vertically upward from over the edge of a building. The object's height (in meters) after t seconds is given by

$$s(t) = -4.9t^2 + 14.7t + 49.$$

(a) Determine the object's maximum height.

$$s'(t) = -9.8t + 14.7 \qquad \boxed{s(1.5) = 60.025 \text{ FT}}$$

$$s'(t) = 0 \Rightarrow t = \frac{14.7}{9.8} = 1.5$$

(b) What is the object's speed when it hits the ground?

$$s(t) = 0 \Rightarrow -4.9t^2 + 14.7t + 49 = 0$$

$$-4.9(t^2 - 3t - 10) = 0 \Rightarrow -4.9(t-5)(t+2) = 0$$

$$t = 5$$

$$|s'(5)| = |-9.8(5) + 14.7| = \boxed{34.3 \text{ m/sec}}$$

8. Find the derivative of $f(x) = x^2\sqrt{1 - x^2}$.

$$\boxed{f'(x) = 2x\sqrt{1-x^2} + x^2\left(\frac{1}{2}\right)(1-x^2)^{-1/2}(-2x) = 2x\sqrt{1-x^2} - \frac{x^3}{\sqrt{1-x^2}}}$$

9. Find the derivative of $R(x) = \left(\frac{3x - 1}{x^2 + 3}\right)^5$.

$$\boxed{R'(x) = 5\left(\frac{3x-1}{x^2+3}\right)^4 \left(\frac{3(x^2+3) - 2x(3x-1)}{(x^2+3)^2}\right)}$$