Math 131 - Assignment 7

March 20, 2024

Name Key

Score ____________

Show all work to receive full credit. Supply explanations when necessary. Use extra paper as necessary. This assignment is due March 27.

1. Suppose f and f^{-1} are differentiable functions. The table below shows the values of $f(x)$ and $f'(x)$ at selected values of x. Find $(f^{-1})'(3)$. Show how you got it.

x	0	1	2	3
$f(x)$	5	3	7	2
$f'(x)$	0	8	4	1

$f^{-1}(3) = 1$ BECAUSE $f(1) = 3$.

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(1)} = \boxed{\frac{1}{8}}$$

2. Let $g(x) = (\cos^{-1} x)^2$. Find the **exact value** of $g'(1/2)$. Simplify your answer as much as possible.

$$g'(x) = 2\cos^{-1}(x)\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$g'(1/2) = 2\cos^{-1}(1/2)\left(\frac{-1}{\sqrt{1-1/4}}\right) = 2\left(\frac{\pi}{3}\right)\left(\frac{-1}{\sqrt{3/4}}\right) = 2\left(\frac{\pi}{3}\right)\left(\frac{-2}{\sqrt{3}}\right)$$

$$= \boxed{\frac{-4\pi}{3\sqrt{3}}}$$

3. Find $h'(x)$ if $h(x) = \log_5[(6x^2+4)^9]$.

$$h(x) = \frac{9}{\ln 5}\ln(6x^2+4) \Rightarrow \boxed{h'(x) = \frac{9}{\ln 5}\left(\frac{12x}{6x^2+4}\right)}$$

4. Find dy/dx if $y = x^3 e^{\cot x}$.

$$\boxed{\frac{dy}{dx} = 3x^2 e^{\cot x} + x^3 e^{\cot x}(-\csc^2 x)}$$

5. Use logarithmic differentiation to find dy/dx when $y = (\sin x)^x$.

$$\ln y = x \ln \sin x$$

$$\frac{1}{y}\frac{dy}{dx} = \ln \sin x + (x)\left(\frac{\cos x}{\sin x}\right)$$

$$= \ln \sin x + x \cot x$$

$$\boxed{\frac{dy}{dx} = (\sin x)^x\left(\ln(\sin x) + x\cot x\right)}$$

6. Let $f(x) = \ln(x^2+1)$. Find $f'(x)$ and use it to determine a point at which the graph's tangent line is horizontal.

$f'(x) = \dfrac{2x}{x^2+1}$ $\quad f'(x) = 0 \Rightarrow 2x = 0 \Rightarrow \underbrace{x = 0 \Rightarrow y = \ln(1) = 0}$

$\boxed{\text{POINT IS } (0,0)}$

7. Find $g''(x)$ if $g(x) = e^{-5x^2}$.

$g'(x) = -10x\, e^{-5x^2}$

$g''(x) = -10\, e^{-5x^2} - 10x(-10x)e^{-5x^2} \Rightarrow \boxed{g''(x) = (100x^2 - 10)e^{-5x^2}}$

8. Use logarithmic differentiation to find $\dfrac{dy}{dx}$ when $y = \dfrac{(x+1)^2(x^3+1)}{4x^2(x-5)}$.

$\ln y = 2\ln(x+1) + \ln(x^3+1) - \ln(4x^2) - \ln(x-5)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{2}{x+1} + \dfrac{3x^2}{x^3+1} - \dfrac{8x}{4x^2} - \dfrac{1}{x-5} \Rightarrow \boxed{\dfrac{dy}{dx} = \left(\dfrac{2}{x+1} + \dfrac{3x^2}{x^3+1} - \dfrac{2}{x} - \dfrac{1}{x-5}\right)\left(\dfrac{(x+1)^2(x^3+1)}{4x^2(x-5)}\right)}$

9. Find $\dfrac{dy}{dx}$ if $y = \tan^{-1}(\sqrt{x})$.

$\dfrac{dy}{dx} = \dfrac{1}{1+(\sqrt{x})^2} \cdot \dfrac{1}{2}x^{-1/2} = \boxed{\dfrac{1}{2\sqrt{x}(1+x)}}$

10. Find $f'(x)$ if $f(x) = e^{x \ln x}$.

$f'(x) = e^{x\ln x} \cdot \dfrac{d}{dx}(x \ln x) = e^{x \ln x}\left(\dfrac{x}{x} + \ln x\right) = \boxed{e^{x\ln x}(1 + \ln x)}$

11. Find an equation of the line tangent to the graph of $y = 2^{x^3}$ at the point where $x = 1$.

SLOPE...

$\dfrac{dy}{dx} = 2^{x^3} 3x^2 \ln 2$

$m = \left.\dfrac{dy}{dx}\right|_{x=1} = 2 \cdot 3 \cdot \ln 2$

$= 6 \ln 2$

POINT...

$x = 1 \Rightarrow$

$y = 2$

$(1, 2)$

TAN LINE...

$\boxed{y - 2 = 6\ln 2\,(x-1)}$