Math 131 - Assignment 8

March 27, 2024

Name Key

Score ____________

Show all work to receive full credit. Supply explanations when necessary. Use extra paper as necessary. This assignment is due April 3.

1. A particle is moving along the graph of $y = \sqrt{x}$ in such a way that $\frac{dx}{dt} = 5$. Find $\frac{dy}{dt}$ when $x = 9$.

$$\frac{dy}{dt} = \frac{1}{2} x^{-1/2} \frac{dx}{dt}$$

WHEN $X = 9$

$$\frac{dy}{dt} = \frac{1}{2}(9)^{-1/2}(5) = \boxed{\frac{5}{6}}$$

2. Suppose that the infected region of an injury is circular, and its radius is growing at the rate of 1.2 mm/hr. Find the rate of change of the area of the infected region when the radius is 3.4 mm.

r = RADIUS OF REGION AT TIME t

A = AREA OF REGION AT TIME t

$\frac{dr}{dt} = 1.2$ mm/hr FIND $\frac{dA}{dt}$ WHEN $r = 3.4$ mm.

$$A = \pi r^2$$

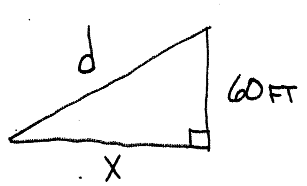
$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

WHEN $r = 3.4$ ...

$$\frac{dA}{dt} = 2\pi(3.4)(1.2) = 8.16\pi \ \frac{mm^2}{hr}$$

≈ 25.6

3. A girl on flat ground flies a kite at a height of 60 ft. The wind carries the kite horizontally away from her at a rate of 5 ft/sec. How fast is the distance (diagonally) between the girl and the kite increasing when the kite is 150 ft away from her?



$\frac{dx}{dt} = 5$ FT/SEC

FIND $\frac{dd}{dt}$ WHEN $d = 150$ FT.

$$X^2 + 3600 = d^2$$

$$2x\frac{dx}{dt} = 2d\frac{dd}{dt}$$

$$\frac{dd}{dt} = \frac{x}{d}\frac{dx}{dt}$$

WHEN $d = 150$ FT

$$X = \sqrt{(150)^2 - 3600} = \sqrt{18{,}900}$$

AND

$$\frac{dd}{dt} = \frac{\sqrt{18900}}{150}(5)$$

$$= \boxed{\frac{\sqrt{189}}{3} \ \text{FT/SEC}} \approx 45.8$$

4. Let $f(x) = \frac{1}{x} + \sqrt[3]{x}$. $f'(x) = -\frac{1}{x^2} + \frac{1}{3}x^{-2/3}$

(a) Determine the linearization of f at $x = 8$. Write your answer in exact form (fractions, not decimals).

$f(8) = \frac{1}{8} + 2 = \frac{17}{8}$

$f'(8) = -\frac{1}{64} + \frac{1}{3(4)} =$

$-\frac{1}{64} + \frac{1}{12} = \frac{13}{192}$

$$\boxed{L(x) = \frac{17}{8} + \frac{13}{192}(x-8)}$$

(b) Use your linearization to approximate $f(8.1)$. Round to the 6th decimal place.

$f(8.1) \approx L(8.1) = \frac{17}{8} + \frac{13}{192}(0.1) = \frac{4093}{1920}$

≈ 2.131771

5. Some values of $f(x)$ and $f'(x)$ near $x = 1$ are given in the table below.

x	0.50	0.75	1.00	1.25	1.50
$f(x)$	6.08	6.90	8.00	9.41	11.14
$f'(x)$	2.74	3.82	5.00	6.26	7.60

(a) Determine the linearization of f at $x = 1$.

$L(x) = f(1) + f'(1)(x-1)$

$$\boxed{L(x) = 8 + 5(x-1)}$$

(b) Use the linearization you found above to approximate $f(0.75)$.

$f(0.75) \approx L(0.75) = 8 + 5(-0.25)$

$= \boxed{6.75}$

6. Find the linearization of $f(x) = x^2 + x^{1/2} + \dfrac{1}{x}$ at $x = 1$, then use it to approximate $f(0.98)$.

$f(1) = 3$

$\boxed{L(x) = 3 + \frac{3}{2}(x-1)}$

$f'(x) = 2x + \frac{1}{2}x^{-1/2} - \frac{1}{x^2}$

$f(0.98) \approx L(0.98) = 3 + \frac{3}{2}(-0.02)$

$f'(1) = 2 + \frac{1}{2} - 1 = \frac{3}{2}$

$= \boxed{2.97}$

7. Use differentials to approximate the change in $y = \sqrt{x^3+1}$ as x changes from 2 to 2.07.

$dy = \frac{1}{2}(x^3+1)^{-1/2}(3x^2)\,dx$

$\Delta y \approx \frac{1}{2}(9)^{-1/2}(12)(0.07) = \boxed{0.14}$

8. Determine the differential dy.

(a) $y = 5^{x^2+1}$

$\boxed{dy = 5^{x^2+1}(\ln 5)(2x)\,dx}$

(b) $y = \cot^{-1}(\sqrt{x})$

$dy = \dfrac{-1}{1+x}\,\dfrac{1}{2\sqrt{x}}\,dx$

$= \boxed{\dfrac{-1}{2\sqrt{x}(1+x)}}$

9. Use differentials to approximate the change in $y = \dfrac{1}{1-x}$ as x changes from 2 to 1.98.

$dy = \dfrac{1}{(1-x)^2}\,dx$

$\Delta y \approx \dfrac{1}{(1-2)^2}(-0.02) = \boxed{-0.02}$

10. Suppose that the percent error in measuring the side length of a cube is 2%. Use differentials to estimate the percent error in computing the cube's volume.

$V = s^3$

$dV = 3s^2\,ds$

$\Delta s = 0.02s$

$\Delta V \approx 3s^2(0.02s) = 0.06s^3 = 0.06V$

$= \boxed{6\% \text{ OF } V}$