Math 131 - Assignment 9

April 3, 2024

Name Key

Score ____________

Show all work to receive full credit. Supply explanations when necessary. Use extra paper as necessary. This assignment is due April 10.

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1. Find the absolute maximum and minimum values of $h(x) = x^3 - 3x^2 + 1$ on $[-1/2, 4]$.

$h'(x) = 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0,\ x = 2$

CRIT #'s : $x = 0,\ x = 2$

END PTS : $x = -1/2,\ x = 4$

x	$h(x)$
0	1
2	$\boxed{-3 \leftarrow \text{ABS MIN}}$
$-1/2$	$1/8$
4	$\boxed{17 \leftarrow \text{ABS MAX}}$

2. Find the critical numbers of $f(x) = x^4 + 4x^3 - 36x^2$.

$f'(x) = 4x^3 + 12x^2 - 72x$

$= 4x(x^2 + 3x - 18)$

$= 4x(x+6)(x-3)$

$f'(x)$ DNE NOWHERE

CRIT #'s ARE $x = -6,\ x = 0,\ x = 3$

$f'(x) = 0 \Rightarrow x = 0,\ x = -6,\ x = 3$

3. Find the absolute extreme values of $f(x) = x - 2\sin x$ on $[-2, 3]$.

$f'(x) = 1 - 2\cos x$

$f'(x) = 0 \Rightarrow \cos x = \frac{1}{2}$

$$\underbrace{\begin{array}{l} x = \frac{\pi}{3} + 2k\pi \\ x = -\frac{\pi}{3} + 2k\pi \end{array}}$$

THE ONLY SOLUTIONS IN $[-2, 3]$ HAVE $k = 0$.

CRIT #'s ARE $\frac{\pi}{3}, -\frac{\pi}{3}$

ENDPTS : $x = -2,\ x = 3$

x	$f(x)$
$\frac{\pi}{3}$	$\boxed{-0.68485 \leftarrow \text{ABS MIN}}$
$-\frac{\pi}{3}$	0.68485
-2	-0.18141
3	$\boxed{2.71776 \leftarrow \text{ABS MAX}}$

4. Find all critical numbers of $f(x) = 5x^{3/7} - 2x^{10/7}$. ← DOMAIN IS ALL REAL #'S : $\mathbb{R}$
(Helpful hint: Simplify your derivative by factoring out $x^{-4/7}$.)

$$f'(x) = \frac{15}{7}x^{-4/7} - \frac{20}{7}x^{3/7}$$

$$= \frac{1}{7}x^{-4/7}(15-20x) = \frac{15-20x}{7x^{4/7}}$$

$f'(x)$ DNE WHEN $x^{4/7}=0 \Rightarrow x=0$

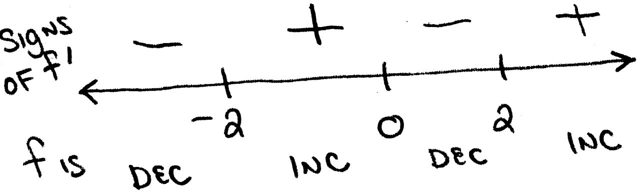
$f'(x)=0$ WHEN $15-20x=0 \Rightarrow x=3/4$

CRIT #'S ARE
$x=0$,
$x=3/4$

5. Find open intervals on which the graph of $f(x) = 2x^4 - 16x^2 + 3$ is increasing/decreasing. Also identify all relative extreme values.

$$f'(x) = 8x^3 - 32x = 8x(x^2-4) = 8x(x-2)(x+2)$$

$f'(x)=0 \Rightarrow x=0,\ x=2,\ x=-2$



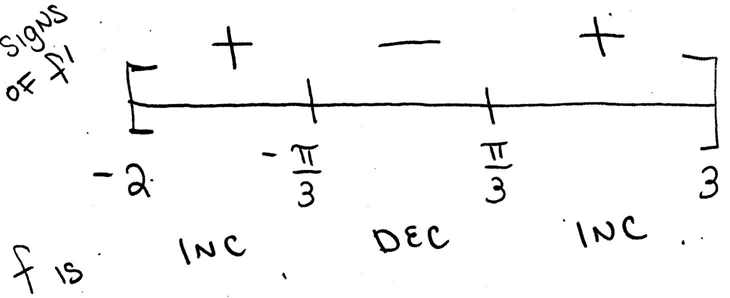
f IS INCREASING ON $(-2,0) \cup (2,\infty)$
f IS DECREASING ON $(-\infty,-2) \cup (0,2)$

$f(-2) = -29$ IS A REL MIN
$f(0) = 3$ IS A REL MAX
$f(2) = -29$ IS A REL MIN

6. Let $f(x) = x - 2\sin x$ on $[-2, 3]$. Find open intervals on which the graph of f is increasing/decreasing. Also identify all relative extreme values.

SEE PROB #3

$f'(x) = 1 - 2\cos x$



f IS INCREASING ON $(-2, -\frac{\pi}{3}) \cup (\frac{\pi}{3}, 3)$
f IS DECREASING ON $(-\frac{\pi}{3}, \frac{\pi}{3})$

$f(-\frac{\pi}{3}) = -\frac{\pi}{3} + \sqrt{3}$ IS A REL MAX.

$f(\frac{\pi}{3}) = \frac{\pi}{3} - \sqrt{3}$ IS A REL MIN.

7. Let $g(x) = x^4 + \cos(20x)$. Without looking at the graph of g, determine whether the graph is concave up or concave down at the point where $x = 0.7$.

$$g'(x) = 4x^3 - 20\sin(20x)$$

$$g''(x) = 12x^2 - 400\cos(20x)$$

$g''(0.7) = -48.8149 \Rightarrow$ GRAPH IS CONCAVE DOWN.

8. Let $f(x) = (x-6)^3(x-2)$. Find $f''(x)$ and write it in factored form. Then find open intervals on which the graph of f is concave up/down. Identify all points of inflection of the graph of f.

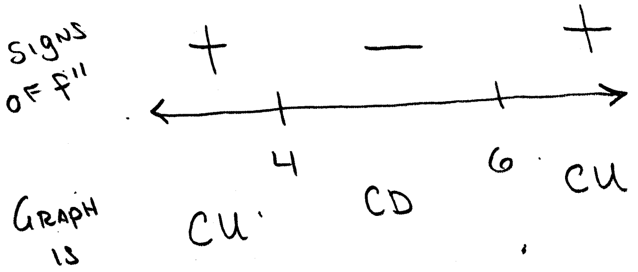
$$f'(x) = 3(x-6)^2(x-2) + (x-6)^3 = (x-6)^2\left[3(x-2) + (x-6)\right]$$

$$= (x-6)^2(4x-12)$$

$$f''(x) = 2(x-6)(4x-12) + (x-6)^2(4) = 4(x-6)(2x-6+x-6)$$

$$= 4(x-6)(3x-12) = 12(x-6)(x-4)$$

$$f''(x) = 0 \Rightarrow x = 6,\ x = 4$$



GRAPH IS CU ON $(-\infty, 4) \cup (6, \infty)$

GRAPH IS CD ON $(4, 6)$

$(4, -16)$ AND $(6, 0)$ ARE INF. PTS