

Math 131 - Test 1
February 7, 2024

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. Determine all limits analytically unless otherwise indicated. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist). When classifying discontinuities, use the words *removable*, *nonremovable*, *jump*, and/or *infinite*.

1. (6 points) Estimate the following limit by using a table of appropriate numerical values. Use a sufficient number of values to feel confident about your result when rounded to three decimal places.

$$\lim_{x \rightarrow 0^+} (1 + x)^{1/x}$$

2. (9 points) These limits DO NOT EXIST. Carefully explain why each limit fails to exist.

(a) $\lim_{x \rightarrow \pi/2} \frac{\tan x}{x}$

(b) $\lim_{x \rightarrow 0} \frac{2x^3 + x}{|x|}$

(c) $\lim_{x \rightarrow 3} \sqrt{9 - x^2}$

3. (24 points) Determine each limit analytically, or explain why the limit does not exist.

$$(a) \lim_{x \rightarrow 20} \frac{x - 20}{\sqrt{x - 4} - 4}$$

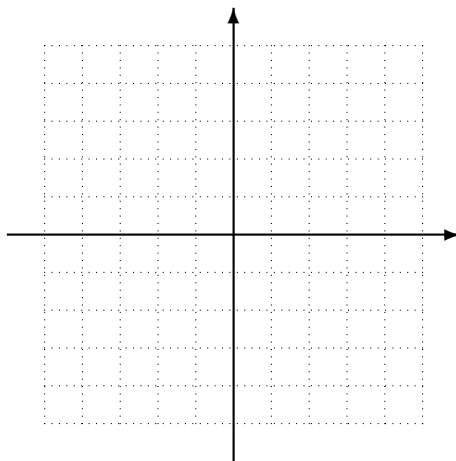
$$(b) \lim_{w \rightarrow -5} \frac{2w^2 + 10w}{w^2 - w - 30}$$

$$(c) \lim_{x \rightarrow 2} \frac{\frac{1}{2} - \frac{1}{x}}{x - 2}$$

$$(d) \lim_{r \rightarrow -2} \frac{r^2 + 6r + 8}{r^2 + 4}$$

4. (7 points) Sketch the graph of a function f such that

- f is defined for all real numbers between -5 and 5 ,
- $\lim_{x \rightarrow 0} f(x) = 2$,
- $f(0) = 0$,
- $\lim_{x \rightarrow 3^-} f(x) = \infty$,
- $\lim_{x \rightarrow 3^+} f(x) = -1$, and
- $f(3) = 2$.



5. (10 points) Consider the function $f(x) = \begin{cases} x^2 + x + 2, & x < 0 \\ \frac{\sin 4x}{2x}, & x > 0 \end{cases}$

(a) Evaluate the limit: $\lim_{x \rightarrow \pi} f(x)$.

(b) Evaluate the limit: $\lim_{x \rightarrow 0^+} f(x)$.

(c) Evaluate the limit: $\lim_{x \rightarrow 0^-} f(x)$.

(d) Is f continuous at $x = 0$? Explain why or why not.

(e) If your answer to part (d) was “no,” then classify the discontinuity. Otherwise, draw a smiley face.

6. (4 points) Give an example of a function with a removable discontinuity at $x = 3$ and an infinite discontinuity at $x = 1$.

7. (12 points) In each part below, determine analytically whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

(a) $\lim_{x \rightarrow -4} \frac{x + 5}{(x + 4)^2}$

(b) $\lim_{x \rightarrow -4^-} \frac{x + 5}{x + 4}$

(c) $\lim_{x \rightarrow -4^+} \frac{x + 5}{x + 4}$

(d) $\lim_{x \rightarrow -4} \frac{x + 5}{x + 4}$

8. (4 points) Use the limit laws to rewrite the limit in terms of only limits of x and limits of constants. Then give the value of the limit.

$$\lim_{x \rightarrow 1} (x^2 + 5x - 8)$$

9. (4 points) Suppose $\lim_{x \rightarrow 7} f(x) = 2$, $\lim_{x \rightarrow 7} h(x) = 2$, and $f(x) \leq g(x) \leq h(x)$ for all x . What can you say about $\lim_{x \rightarrow 7} g(x)$? Explain your reasoning.

10. (5 points) Show that g is not continuous at $x = 5$.

$$g(x) = \begin{cases} \sqrt{x^2 + 2x + 1}, & x < 5 \\ 6, & x = 5 \\ x + 1 - \cos \pi x, & x > 5 \end{cases}$$

11. (5 points) Indicate whether each statement is true (T) or false (F).

(a) _____ A jump discontinuity might also be a removable discontinuity.

(b) _____ $\lim_{x \rightarrow 0} \sqrt{x} = 0$

(c) _____ The limit of a polynomial function can always be found by direct substitution.

(d) _____ If $\lim_{x \rightarrow 2} f(x) = f(2)$, then f is continuous at $x = 2$.

(e) _____ The limit of a rational function can always be found by direct substitution.

12. (2 points) Suppose you were asked to use a table of values to estimate $\lim_{x \rightarrow 1} f(x)$. Which list of x -values shown below would be best for your table?
- (a) $x = 0.9, 0.99, 0.999, 1.1, 1.11, 1.111$
 - (b) $x = 0.9, 0.99, 0.999, 1.0, 1.1, 1.01, 1.001$
 - (c) $x = 0.9, 0.99, 0.999, 1.1, 1.01, 1.001$
 - (d) $x = 1.00001, 1.000001, 1.0000001, 1.00000001, 1.000000001$
13. (2 points) Which one of the following best describes the meaning of the statement $\lim_{x \rightarrow 0} g(x) = \infty$?
- (a) Direct substitution results in division by zero.
 - (b) The limit at $x = 0$ exists, and it is a very large positive number.
 - (c) The limit at $x = 0$ does not exist because $g(0)$ is not defined.
 - (d) The limit at $x = 0$ does not exist because the values of g grow positively without bound as $x \rightarrow 0$.
14. (2 points) Which of these IS a reason that a limit may not exist?
- (a) The function is not defined to the right of the limit point.
 - (b) The function is not defined at the limit point.
 - (c) Direct substitution cannot be applied.
 - (d) The function is not continuous at the limit point.
15. (2 points) Suppose $\lim_{x \rightarrow c} f(x) = \infty$. Which one of the following is NOT necessarily true?
- (a) The graph of f has a vertical asymptote at $x = c$.
 - (b) $\lim_{x \rightarrow c^+} f(x) = \infty$
 - (c) $\lim_{x \rightarrow c^-} f(x) = \infty$
 - (d) f is not defined at $x = c$.
16. (2 points) Suppose $\lim_{x \rightarrow 3} f(x) = 10$. Which one of these statements must be true?
- (a) f is continuous at $x = 3$.
 - (b) f is defined at $x = 3$.
 - (c) $f(3) = 10$
 - (d) $\lim_{x \rightarrow 3^+} f(x) = 10$