

Math 131 - Test 2

March 6, 2024

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. Unless otherwise indicated, use differentiation rules for all derivatives and do not simplify.

1. (8 points) We studied three ways in which a derivative can fail to exist. Describe them. Then sketch the graph of a function that illustrates exactly two of the three failures and indicate exactly where those failures occur.

2. (5 points) Suppose f is a function for which $f'(5)$ exists. Which of the following are equal to $f'(5)$? Circle all that apply.

(a) The slope of the line tangent to the graph of f at $x = 5$

(b) The instantaneous rate of change of f at $x = 5$

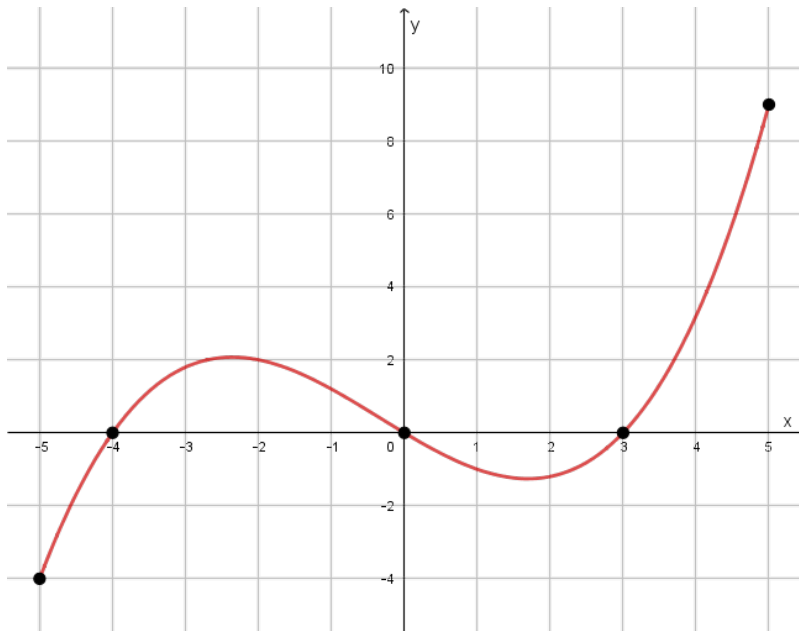
(c) $\lim_{h \rightarrow 0} \frac{f(5+h) - f(5)}{h}$

(d) The slope of the secant line passing through the point where $x = 5$

(e) $\lim_{h \rightarrow 0} \frac{f(5-h) - f(5)}{h}$

3. (10 points) Use the **limit definition of derivative** to find $f'(x)$ when $f(x) = x - 4x^2$. Once you have found $f'(x)$, use it to compute $f'(5)$.

4. (5 points) The graph of f is shown below. Use it to estimate the value of $f'(3)$. Briefly explain your reasoning.



5. (6 points) Use the quotient rule and trig identities to derive our formula for the derivative of $y = \cot x$.

6. (20 points) Determine the derivative of each function. Do not simplify.

(a) $y = 5x^6 - x + 10 + \sqrt[5]{x^3} - \frac{3}{x^4}$

(b) $f(x) = \cos(\pi x^2 + 1)$

(c) $g(t) = t^2 \left(\frac{2}{t^2} - \frac{1}{t^2 + t + 1} \right)$

(d) $f(x) = (\tan x + \sin 2x)^5$

7. (4 points) Suppose $M(t)$ is a differentiable function that gives the mass of a snowball, in kilograms, after t minutes as it grows while rolling down a hill.

(a) What are the units on $M'(t)$?

(b) What are the units on $M''(t)$?

8. (14 points) On Mars, the acceleration due to the force of gravity is about $g = 3.7 \text{ m/s}^2$ (much less than on Earth). In an experiment on the surface of Mars, a probe throws an object straight upward so that its height in meters after t seconds is given by

$$s(t) = -1.85t^2 + 1.3875t + 0.4625.$$

Include units with your answer for each part of this problem. Also, it may be helpful to know that $s(t)$ can be written $s(t) = -0.4625(4t^2 - 3t - 1)$.

(a) Determine the average velocity of the object over the first half second of travel.

(b) When will the object reach its maximum height?

(c) What is the maximum height of the object?

(d) When does the object hit the ground?

(e) What is the object's velocity when it hits the ground?

(f) What is the object's initial height?

(g) What is the overall length of the object's path?

9. (12 points) Assume that y is implicitly defined as a function of x by the equation

$$x^3y^5 + 3x = 8y^3 + 6.$$

(a) Find $\frac{dy}{dx}$.

(b) Find an equation of the line tangent to the graph of the equation at the point $(2, 1)$.

(c) Find an equation of the line normal to the graph of the equation at the point $(2, 1)$.

10. (8 points) Let $F(x) = x^2 \cos x$. Find $F''(x)$.

11. (8 points) Determine the points (both coordinates) on the graph of $y = 2x^3 + 3x^2 - 36x$ at which the tangent line is horizontal. (You should find two such points.)