

**Math 131 - Test 3**  
April 10, 2024

Name \_\_\_\_\_

Score \_\_\_\_\_

**Show all work to receive full credit.** Supply explanations where necessary.

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1. (6 points) Let  $f(x) = x^5 + x^3 - 30$ . Find  $f^{-1}(10)$  and then find  $(f^{-1})'(10)$ . (You must show work to get points.)

(a)  $f^{-1}(10) = 2$  and  $(f^{-1})'(10) = \frac{1}{92}$

(b)  $f^{-1}(10) = 100970$  and  $(f^{-1})'(10) = \frac{1}{50300}$

(c)  $f^{-1}(10) = 30$  and  $(f^{-1})'(10) = 50300$

(d)  $f^{-1}(10) = 2$  and  $(f^{-1})'(10) = \frac{1}{40}$

2. (4 points) Let  $h(x) = \sin^{-1}(f(x))$ . With the information below, compute  $h'(3)$ .

$$f(1) = \frac{1}{3}, \quad f'(1) = \frac{\sqrt{5}}{2}, \quad f(3) = \frac{\sqrt{3}}{2}, \quad f'(3) = \frac{1}{2}$$

3. (6 points) Determine each derivative.

(a)  $\frac{d}{dx} [x \cot^{-1}(x^2)]$

(b)  $\frac{d}{dt} \left( \frac{5}{e^{\sqrt{t}}} \right)$

4. (5 points) Find the slope of the line tangent to the graph of  $y = \log_8(x^3 + x)$  at the point where  $x = 2$ . Write your final answer in decimal form, rounded to the nearest thousandth.

5. (8 points) Use logarithmic differentiation to find  $\frac{dy}{dx}$  when  $y = \frac{2x^5}{\sqrt{x+1}(x^2+1)}$

6. (4 points) Find the instantaneous rate of change of  $g(x) = 2^{3x+1}$  at the point where  $x = 1$ .

7. (6 points) A particle is moving along the graph of  $x^2 + y^3 = 3$  in such a way that  $\frac{dy}{dt} = -8$ . Find  $\frac{dx}{dt}$  when  $x = 2$ .

8. (6 points) A big block of ice is in the shape of a perfect cube. As it melts, the length of each edge of the cube is decreasing at a rate of 2 cm/hr. At what rate is the block's volume changing when the side length is 20 cm?
9. (5 points) Find the linearization of  $f(x) = \tan^{-1} x$  at  $x = 1$ . Then use your linearization to approximate  $f(0.92)$ .
10. (6 points) Let  $y = e^{4x} \cos x$ . Determine the differential  $dy$ . Then use differentials to estimate  $\Delta y$  when  $x$  changes from  $x = 0$  to  $x = 0.94$ .

11. (6 points) Explain what it means to be a critical number for a function  $f$ . Then say what you would look for on the graph of  $f$  if you were trying to use the graph to identify critical numbers.

12. (6 points) Let  $h(x) = x\sqrt{2x+1}$ . Find all  $x$ -values for which  $h'(x) = 0$  or  $h'(x)$  DNE. Then say which of those values are critical numbers.

13. (8 points) Use calculus techniques to find the absolute maximum and minimum values of  $f(x) = 3x^4 + 2x^3 - 3x^2$  on  $[-2, 1]$ .

14. (13 points) Let  $f(x) = \frac{1}{5}x^5 - x^4 - \frac{5}{3}x^3 + 17$ .

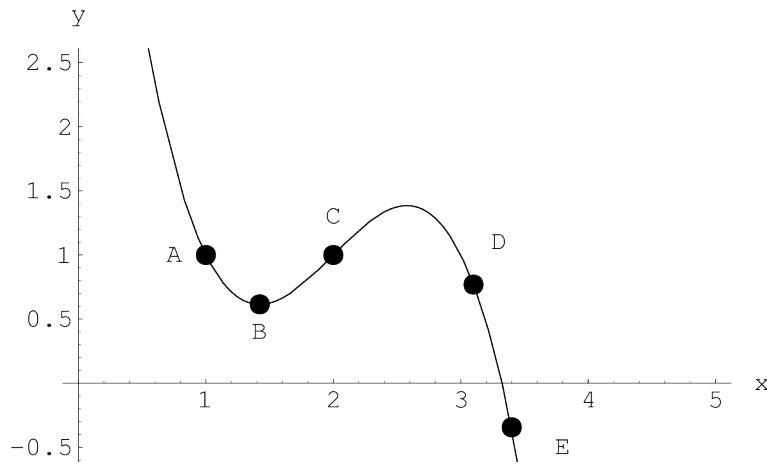
(a) Find the critical numbers of  $f$ .

(b) Use calculus techniques to find open intervals on which  $f$  is increasing/decreasing.

(c) Identify all relative extreme values.

15. (5 points) Use the 2nd derivative to determine whether the graph of  $r(x) = x^3 + \sin(10x)$  is concave up or concave down at the point where  $x = 0.65$ .

16. (6 points) The graph of  $f$  is shown below. For each part of this problem, find a labeled point that satisfies the given condition.



(a)  $f''(x) = 0$

(b)  $f'(x) = 0$

(c)  $f''(x) < 0$

(d)  $f(x) < 0$

(e)  $f'(x) > 0$

(f)  $f''(x) > 0$