Math 131 - Final Exam

May 8, 2024

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

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1. (10 points) Use algebraic techniques (not a graph, table, or L'Hôpital's rule) to determine each limit. You may need to use $+\infty$, $-\infty$, or DNE.

   (a) $\displaystyle \lim_{y \to 0} \frac{y}{(y+6)^2 - 36}$

   (b) $\displaystyle \lim_{x \to 3} \frac{x^2 + 9}{(x^2 - 9)^2}$

2. (10 points) Carefully explain why f is NOT continuous at $x = 4$. Then state the type of discontinuity and further explain how f could be made continuous at that point.

$$f(x) = \begin{cases} x^2 - 5x + 5, & x < 4 \\ 4\cos(\pi x)/x, & x > 4 \end{cases}$$

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3. (10 points) Let $f(x) = 3x - x^2$. Use the **limit definition of the derivative** to determine $f'(x)$. Show all work.

4. (10 points) Use basic differentiation rules to determine each derivative. Do not simplify.

(a) $\dfrac{d}{dx}\left(\dfrac{e^{2x}}{x^2 + 1}\right)$

(b) $\dfrac{d}{dw}\sin^{-1}(\sqrt{w})$

5. (10 points) Use logarithmic differentiation to compute dy/dx.

$$y = \frac{x^2 + 1}{x^2(x + 3)}$$

6. (10 points) Two cars are driving away from an intersection, one heading due east and the other heading due north. When both cars are 2 miles from the intersection, the eastbound car is traveling at $50\,$mph and the distance between the cars is increasing at $80\,$mph. Find the speed of the northbound car.

7. (10 points) Let $f(x) = x^4 - 4x^3$.

   (a) Use the 1st derivative test to determine all relative extreme values.

   (b) Use the 2nd derivative test to determine open intervals on which the graph of f is concave up/down.

   (c) Draw a rough sketch of the graph of f being sure that your graph shows the features you've found above.

8. (10 points) Use any analytical method (not a table or graph) to determine each limit.

(a) $\lim\limits_{x \to \infty} 5xe^{-x}$

(b) $\lim\limits_{x \to 0} \left(\dfrac{x^2 - 2 + 2\cos x}{x^4} \right)$

9. (10 points) For each part of this problem, set up and evaluate an appropriate definite integral.

(a) Find the average value of $f(x) = \dfrac{2}{x} + \sqrt[4]{x}$ on $[1, 16]$.

(b) Find the area of the fourth quadrant region between the x-axis and the graph of $y = x^2 - 5x + 6$.

10. (10 points) Evaluate each integral. (You may need to use a substitution.)

(a) $\displaystyle\int \left(e^x + \frac{1}{1 + x^2} + \sec^2 x \right) \, dx$

(b) $\displaystyle\int_0^\pi \cos^4 x \, \sin x \, dx$