Math 131 - Test 2
March 11, 2026

Name key

Score

Show all work to receive full credit. Supply explanations where necessary. Unless otherwise indicated, use differentiation rules for all derivatives, and do not simplify.

1. (12 points) The function f has only three possible points of discontinuity: $x = 1$, $x = 2$, and $x = 4$. Analyze the function at each point to determine whether f is continuous. Classify each discontinuity. **Your work must show how you are using limits.**

$$f(x) = \begin{cases} x^2 + 2x + 3, & x < 1 \\ x^3 + 5, & 1 \le x < 2 \\ 6x + \cos(\pi x), & 2 < x \le 4 \\ x + 8\sqrt{x}, & x > 4 \end{cases}$$

$X = 1$

$f(1) = (1)^3 + 5 = 6$

$\lim_{x \to 1^-} f(x) = (1)^2 + 2(1) + 3 = 6$

$\lim_{x \to 1^+} f(x) = (1)^3 + 5 = 6$

$\boxed{f \text{ IS CONTINUOUS AT } X = 1}$

$X = 2$

$f(2)$ IS NOT DEFINED.

$\lim_{x \to 2^-} f(x) = (2)^3 + 5 = 13$

$\lim_{x \to 2^+} f(x) = 6(2) + \cos 2\pi = 13$

$\boxed{X = 2 \text{ IS A REMOVABLE DISCONT.}}$

$X = 4$

$f(4) = 6(4) + \cos 4\pi = 25$

$\lim_{x \to 4^-} f(x) = 6(4) + \cos 4\pi = 25$

$\lim_{x \to 4^+} f(x) = 4 + 8\sqrt{4} = 20$

$\boxed{X = 4 \text{ IS A JUMP DISCONT.}}$

2. (4 points) The function $g(x) = x^2 + 3\sin x$ is continuous everywhere. Use the intermediate value theorem to find an interval on which the equation $g(x) = 4$ has a solution. Briefly explain your reasoning.

$g(1) \approx 3.52 < 4$

$g(2) \approx 6.73 > 4$

BY THE IVT, g WILL TAKE ON ALL VALUES BETWEEN 3.52 AND 6.73 ON THE INTERVAL $[1, 2]$, THIS INCLUDES TAKING ON THE VALUE $g(x) = 4$.

3. (10 points) Let $f(x) = x^2 - 6x + 2$. Use a **limit definition** of the derivative to determine $f'(x)$. Show all work.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[(x+h)^2 - 6(x+h) + 2] - [x^2 - 6x + 2]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{6x} - 6h + \cancel{2} - \cancel{x^2} + \cancel{6x} - \cancel{2}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 6h}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x + h - 6)}{\cancel{h}}$$

$$= \lim_{h \to 0} (2x + h - 6)$$

$$= \boxed{2x - 6}$$

4. (5 points) Use differentiation rules to confirm your derivative above. Then find an equation of the line tangent to the graph of $f(x) = x^2 - 6x + 2$ at the point where $x = 3$.

$$\boxed{f'(x) = 2x - 6}$$

POINT: $x = 3$

$y = f(3) = 9 - 18 + 2 = -7$

TANGENT LINE:

$y + 7 = 0(x - 3)$

OR

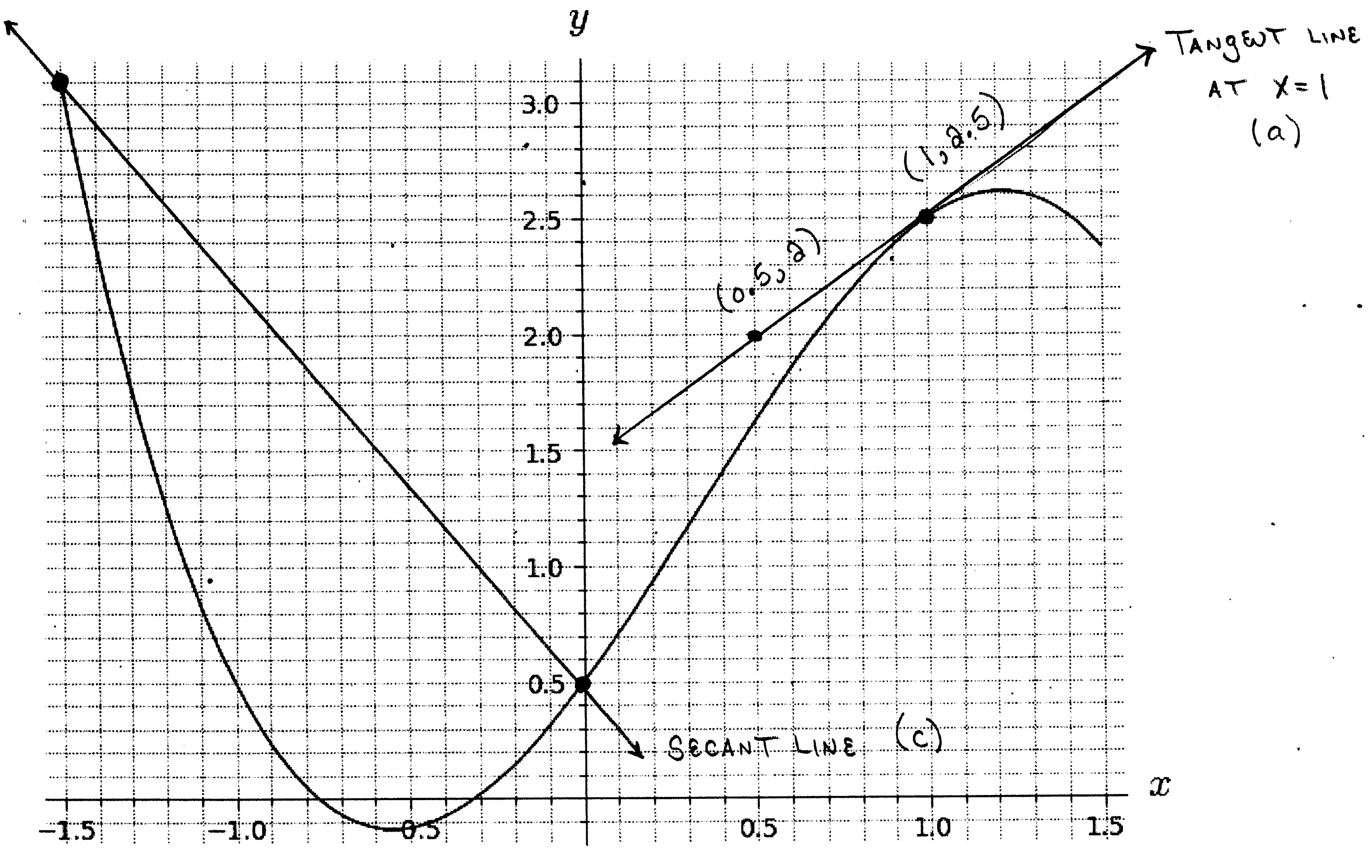
$$\boxed{y = -7}$$

SLOPE:

$m = f'(3) = 2(3) - 6$

$= 0$

5. (10 points) The graph of $y = f(x)$ is shown below. Use the graph for each part of this problem.



(a) Sketch the tangent line at $x = 1$. Then use your tangent line to estimate $f'(1)$. Show work or explain your reasoning.

USING $(1, 2.5)$ AND $(0.5, 2)$, $f'(1) = $ SLOPE OF TANGENT LINE $\approx \frac{\Delta y}{\Delta x} = \frac{2 - 2.5}{0.5 - 1} = \frac{-0.5}{-0.5} = \boxed{1}$

(b) Estimate the interval(s) on which $f'(x) > 0$. Briefly explain your reasoning.

WHEN $f'(x) > 0$,

f IS INCREASING. THIS IS HAPPENING ROUGHLY FROM $X = -0.5$ TO $X = 1.2$

(c) Use a secant line to estimate the average rate of change of f over the interval from $x = -1.5$ to $x = 0$.

USING $(-1.5, 3.1)$ AND $(0, 0.5)$

$$\frac{\Delta y}{\Delta x} = \frac{3.1 - 0.5}{-1.5 - 0} = \frac{2.6}{-1.5} \approx \boxed{-1.73}$$

6. (20 points) Determine the derivative of each function. Show all work. Do not simplify.

(a) $y = 3x^5 - 8x + 12 + x^{7/3} - \frac{5}{x^3}$ $= 3x^5 - 8x + 12 + x^{7/3} - 5x^{-3}$

$$\boxed{\frac{dy}{dx} = 15x^4 - 8 + \frac{7}{3}x^{4/3} + 15x^{-4}}$$

(b) $g(x) = \frac{\cos x}{\sqrt{x}}$

$$\boxed{g'(x) = \frac{-\sqrt{x}\sin x - \frac{1}{2}x^{-1/2}\cos x}{(\sqrt{x})^2}} = \frac{-(2x\sin x + \cos x)}{2(\sqrt{x})^3}$$

(c) $f(t) = \tan(t^2)$

$$\boxed{f'(t) = 2t\sec^2(t^2)}$$

(d) $y = (5x+2)^4(2x+7)^6$

$$\frac{dy}{dx} = \boxed{4(5x+2)^3(5)(2x+7)^6 + (5x+2)^4(6)(2x+7)^5(2)}$$

$$= (5x+2)^3(2x+7)^5\left[20(2x+7) + 12(5x+2)\right]$$

7. (6 points) Let $G(x) = x^6 \sin x$. Find $G''(x)$.

$$G'(x) = 6x^5 \sin x + x^6 \cos x$$

$$G''(x) = 30x^4 \sin x + 6x^5 \cos x + 6x^5 \cos x - x^6 \sin x$$

$$= \boxed{30x^4 \sin x + 12x^5 \cos x - x^6 \sin x}$$

8. An object is launched vertically upward from over the edge of a building. The object's height (in meters) after t seconds is given by

$$s(t) = -4.9t^2 + 14.7t + 49.$$

Include units with your answer for each part of this problem.

(a) (3 points) Determine the average rate of change the object's height over the interval from $t = 0$ to $t = 3$.

$$\frac{\Delta s}{\Delta t} = \frac{s(3) - s(0)}{3-0} = \frac{49-49}{3} = \boxed{0 \text{ m/s}}$$

(b) (3 points) Determine the object's velocity at time $t = 4$.

$$s'(t) = -9.8t + 14.7$$

$$s'(4) = -9.8(4) + 14.7 = \boxed{-24.5 \text{ m/s}}$$

(c) (2 points) What is the acceleration of the object?

$$s''(t) = \boxed{-9.8 \text{ m/s}^2}$$

(d) (4 points) Determine the object's maximum height.

$$s'(t) = 0 \Rightarrow -9.8t + 14.7 = 0$$

$$t = \frac{14.7}{9.8} = 1.5 \text{ s}$$

$$s(1.5) = -4.9(1.5)^2 + 14.7(1.5) + 49 = \boxed{60.025 \text{ m}}$$

(e) (3 points) When does the object hit the ground?

$$s(t) = 0 \Rightarrow -4.9(t^2 - 3t - 10) = 0$$

$$(t-5)(t+2) = 0 \qquad t = \boxed{5 \text{ s}}$$

(f) (1 point) What is the object's initial speed?

$$|v(0)| = |s'(0)| = \boxed{14.7 \text{ m/s}}$$

(g) (1 point) What is the object's speed when it hits the ground?

$$|v(5)| = |-9.8(5) + 14.7| = \boxed{34.3 \text{ m/s}}$$

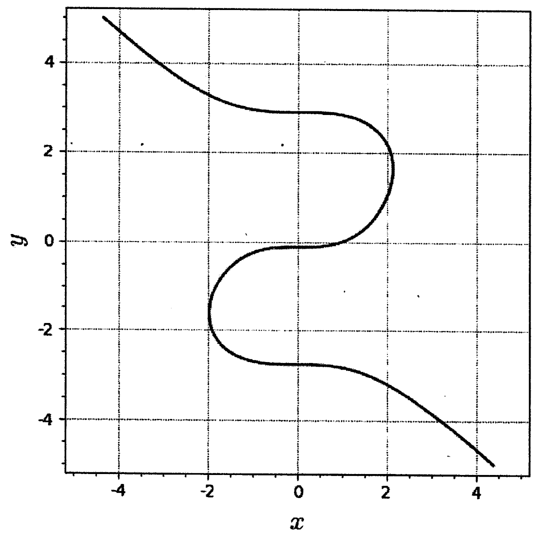
9. (6 points) Suppose the function f is defined for all x. Describe three ways in which $f'(x)$ may fail to exist.

① THE GRAPH OF f HAS A VERTICAL TANGENT LINE AT A POINT. (f' HAS AN INFINITE DISCONT.)

② THE GRAPH OF f HAS A SHARP POINT. (f' HAS A JUMP DISCONT.)

③ f HAS ANY TYPE OF DISCONTINUITY.

10. (10 points) The graph of the equation $x^3 + y^3 = 8y + 1$ is shown below.



(a) Use implicit differentiation to find a formula for dy/dx.

$$\frac{d}{dx}(x^3+y^3) = \frac{d}{dx}(8y+1)$$

$$3x^2 + 3y^2\frac{dy}{dx} = 8\frac{dy}{dx}$$

$$3x^2 = 8\frac{dy}{dy} - 3y^2\frac{dy}{dx}$$

$$3x^2 = (8-3y^2)\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2}{8-3y^2}}$$

(b) Use dy/dx to compute the slope of the graph at the point $(2, 1)$.

$$\left.\frac{dy}{dx}\right|_{(x,y)=(2,1)} = \frac{3(4)}{8-3(1)} = \boxed{\frac{12}{5}}$$