Math 131 - Test 3

Name _____

April 15, 2026

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Let $f(x) = x^2 - 2x - 3$.

   (a) Sketch a rough graph of f. Even though your graph may be rough, clearly indicate the coordinates of the vertex.

   (b) Explain why f does not have an inverse.

   (c) With the restriction to $x \geq 1$, f now has an inverse. With that restriction in mind, compute $f^{-1}(-3)$.

   (d) Find $(f^{-1})'(-3)$.

2. (6 points) Suppose g and g^{-1} are differentiable functions. The table below shows the values of $g(x)$ and $g'(x)$ at selected values of x. Find $(g^{-1})'(5)$. Show how you got it.

x	2	3	4	5
$g(x)$	3	5	6	9
$g'(x)$	2	1	3	7

3. (6 points) Let $g(x) = 4x^2 \sin^{-1} x$. Find $g'(x)$. Then find the exact value of $g'(1/2)$. (Do not give a decimal number for your answer.)

4. (6 points) Find an equation of the line tangent to the graph of $y = \ln(x^2 e^{x+3})$ at the point where $x = 1$.

5. (6 points) Find the instantaneous rate of change of $h(x) = \dfrac{2}{e^{2x} + e^{-2x}}$ at the point where $x = 0$.

6. (2 points) Use the change-of-base formula to rewrite $\log_7 6$ in terms of natural logarithms.

7. (5 points) Determine the derivative of $g(x) = 5^{\tan x}$.

8. (8 points) Use logarithmic differentiation to find dy/dx when $y = \dfrac{\sqrt{x^2+1}}{x^5(8x+7)^2}$.

9. (8 points) Determine the linearization of $f(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}}$ at $x = 4$. Then use your linearization to approximate $f(4.05)$.

10. (6 points) Use differentials to approximate the change in $f(x) = \tan^{-1} x$ as x changes from $x = 1$ to $x = 0.96$.

11. (6 points) Recall that a linearization is essentially a tangent line approximation. It might be useful to think graphically for the following problems.

    (a) Briefly explain why the linearization of $f(x) = e^x$ at $x = 0$ WOULD NOT BE USEFUL for approximating $f(5.1)$.

    (b) Briefly explain why the linearization of $f(x) = 2x+1$ at $x = 0$ WOULD BE USEFUL for approximating $f(5.1)$.

12. (6 points) The length of the base of a right triangle is x cm. Its height is 5 cm more than its base, so that its area is given by

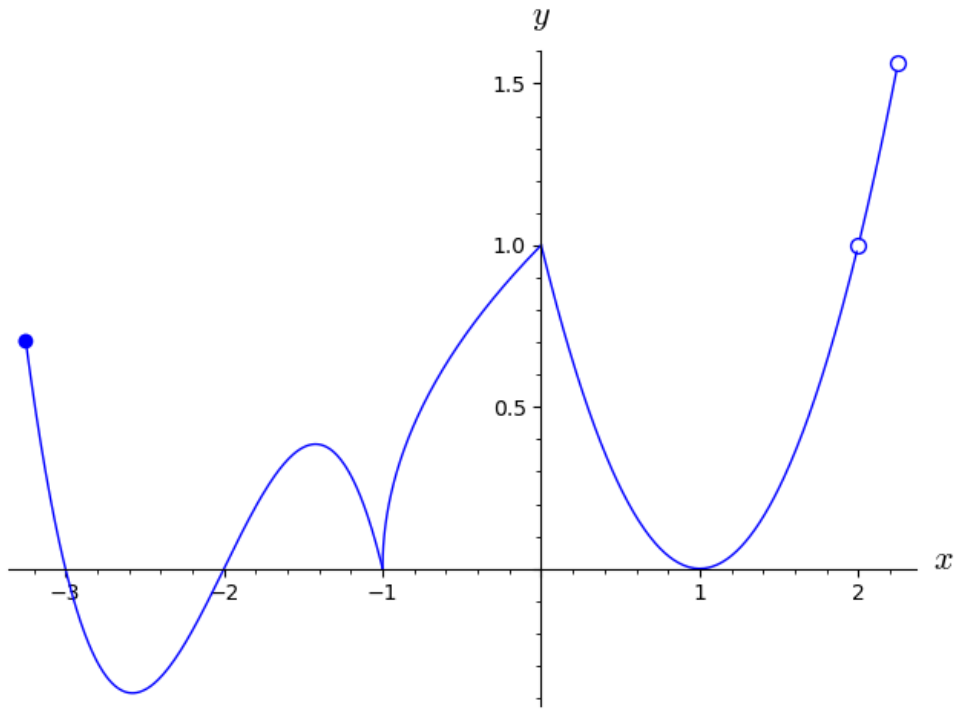
$$A(x) = \frac{1}{2}x(x+5).$$

Use differentials to approximate the change in the triangle's area when $x = 4$ cm and $\Delta x = 0.2$ cm. Include units with your answer.

13. (4 points) Let $f(x) = \ln x$. Find $f'(x)$ and then explain why $x = 0$ IS NOT a critical number of f.

14. (8 points) Find the critical numbers of $f(x) = \frac{1}{4}x^4 - x^3 + x^2 + 1$.

15. (13 points) The graph of g is shown below. Use the graph to solve the problems below. Notice that the domain of g is $[-3.25, 2) \cup (2, 2.25)$.



(a) Estimate the absolute minimum value of g and say where it occurs.

(b) Estimate a relative maximum value of g and say where it occurs.

(c) Explain why g has no absolute maximum value.

(d) Estimate five critical numbers.

(e) Explain why a relative maximum does not occur at the graph's left endpoint.

(f) Estimate a relative minimum value of g and say where it occurs.