

Math 131 - Final Exam
Spring 2026

Name _____
Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Use algebraic techniques (not a graph, table, or L'Hôpital's rule) to determine each limit.

(a) $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{2x^2 - 2}$

(b) $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3}$

2. (10 points) Each function below has a single discontinuity. For each function, find the point of discontinuity and state the kind of discontinuity (removable, jump, or infinite). For full credit, you must show work.

(a) $g(x) = \frac{3x^2 + x}{|x|}$

(b) $h(x) = \frac{\sin(x - 3)}{x - 2}$

(c) $f(x) = \begin{cases} x^2 + 1, & x < 5 \\ 4x + 6, & x > 5 \end{cases}$

3. (10 points) An object is launched vertically upward from over the edge of a cliff. The object's height (in feet) after t seconds is given by

$$s(t) = -16t^2 + 64t + 192.$$

- (a) What is the object's the maximum height? Give units with your answer.
- (b) What is object's velocity when it hits the ground? Give units with your answer.

4. (10 points) Find the slope of the graph of the equation $x^3y^3 - y = x - 9$ at the point $(2, -1)$.

5. (10 points) Let $f(x) = 7 + 2x^2 - 2x^3$.

(a) Use the 1st derivative test to determine all relative extreme values.

(b) Use the 2nd derivative test to determine open intervals on which the graph of f is concave up/down.

6. (10 points) Use any analytical method (not a table or graph) to determine each limit.

(a) $\lim_{x \rightarrow 0^+} \frac{e^x - (1 + x)}{x^3}$

(b) $\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right)$

7. (10 points) Let $f(x) = \frac{1}{x}$, $1 \leq x \leq 2$.

(a) Use 5 subintervals of equal length and subinterval midpoints to compute the corresponding Riemann sum for f over $[1, 2]$.

(b) Sketch the graph of f over the interval $[1, 2]$ and then sketch (in detail) the rectangles associated with your Riemann sum.

(c) Use the Fundamental Theorem of Calculus to briefly explain why your answer in part (a) should be close to $\ln 2$.

8. (10 points) Evaluate each indefinite integral. (Be sure to check your answer by differentiation.)

(a) $\int \left(6x^3 + \sqrt{x} + \frac{5}{x^2} \right) dx$

(b) $\int (e^{4x} + \sec^2 x) dx$

9. (10 points) Evaluate each indefinite integral by using the given substitution.

(a) $\int x \cos(2x^2) dx$, use $u = 2x^2$

(b) $\int e^{\sin x} \cos x dx$, use $u = \sin x$

10. (10 points) Sketch the graph of $y = x^2 - x^3$ carefully showing the bounded region in the first quadrant between the graph and the x -axis. Then use a definite integral to find the area of that bounded region.