Math 132 - Integration Review/Practice

February 25, 2020

1. $\displaystyle\int \frac{2x-9}{\sqrt{x^2-9x+1}}\,dx$

$u = x^2 - 9x + 1$

$du = (2x-9)\,dx$

$$\int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du = 2u^{1/2} + C = 2\sqrt{x^2-9x+1} + C$$

2. $\displaystyle\int \frac{dx}{\sqrt{8x-x^2}}$

COMPLETE THE SQUARE : $-(x^2-8x) = -(x-4)^2 + 16$

$$\int \frac{dx}{\sqrt{16-(x-4)^2}} = \int \frac{du}{\sqrt{16-u^2}} = \sin^{-1}\frac{u}{4} + C$$

$$= \sin^{-1}\left(\frac{x-4}{4}\right) + C$$

$u = x - 4$

$du = dx$

3. $\displaystyle\int (\sec x + \tan x)^2\,dx$

$\sec^2 x - 1$

$$= \int (\sec^2 x + 2\sec x \tan x + \tan^2 x)\,dx$$

$$= \int \sec^2 x\,dx + 2\int \sec x \tan x\,dx + \int (\sec^2 x - 1)\,dx$$

$$= \tan x + 2\sec x + \tan x - x + C$$

4. $\int \frac{3x^2 - 7x}{3x+2}\,dx$

$$\begin{array}{r} x - 3 \\ 3x+2\,\overline{)\,3x^2 - 7x} \\ -(3x^2+2x) \\ \hline -9x \\ -(-9x-6) \\ \hline 6 \end{array}$$

$$= \int (x-3)\,dx + \int \frac{6}{3x+2}\,dx = \boxed{\frac{x^2}{2} - 3x + 6\left(\tfrac{1}{3}\ln|3x+2|\right) + C}$$

5. $\int \frac{3x+2}{\sqrt{1-x^2}}\,dx = \int \frac{3x}{\sqrt{1-x^2}}\,dx + \int \frac{2}{\sqrt{1-x^2}}\,dx$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$$= -\frac{1}{2}\int \frac{3\,du}{\sqrt{u}} + \int \frac{2}{\sqrt{1-x^2}}\,dx$$

$$= -3u^{1/2} + 2\sin^{-1}x + C$$

$$= \boxed{-3\sqrt{1-x^2} + 2\sin^{-1}x + C}$$

6. $\int \frac{e^x}{1+e^x}\,dx = \int \frac{1}{u}\,du$

$u = 1+e^x$

$du = e^x\,du$

$$= \ln|u| + C = \boxed{\ln(1+e^x) + C}$$

7. $\int x\left(1+\frac{1}{x}\right)^3 dx = \int x\left(\frac{x+1}{x}\right)^3 dx = \int \frac{1}{x^2}(x+1)^3\,dx$

$$= \int \frac{1}{x^2}(x^3+3x^2+3x+1)\,dx = \int \left(x + 3 + \frac{3}{x} + \frac{1}{x^2}\right) dx$$

$$= \boxed{\frac{1}{2}x^2 + 3x + 3\ln|x| - \frac{1}{x} + C}$$