Math 132 - Quiz 10

November 9, 2022

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due November 14.

1. (7.5 points) Determine whether the series converges or diverges. Show all work to justify your conclusion.

(a) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) = \sum_{n=1}^{\infty} \left[\ln n - \ln(n+1)\right] = (\ln 1 - \ln 2) + (\ln 2 - \ln 3) + \dots$

TELESCOPING SERIES.

$S_n = \ln 1 - \ln(n+1) = -\ln(n+1)$

$S_n \to -\infty$ SERIES DIVERGES.

(b) $\sum_{n=1}^{\infty} \frac{n^2}{n^2+16}$

$\lim_{n\to\infty} \frac{n^2}{n^2+16} = \lim_{n\to\infty} \frac{1}{1+\frac{16}{n^2}} = 1$

n^{TH} TERM TEST

SERIES DIVERGES BY n^{TH} TERM TEST.

(c) $\sum_{n=1}^{\infty} \frac{3^n+4^n}{7^n} = \underbrace{\sum_{n=1}^{\infty} \left(\frac{3}{7}\right)^n + \sum_{n=1}^{\infty} \left(\frac{4}{7}\right)^n}_{\text{GEOMETRIC.}}$

$r = \frac{3}{7}$ AND $r = \frac{4}{7}$

GEOMETRIC

SERIES CONVERGES.

Turn over.

(d) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ $= \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{1/2}}$

THIS IS A LOGARITHMIC p-SERIES WITH $p = \frac{1}{2} < 1$.

IT $\boxed{\text{DIVERGES.}}$

(e) $\sum_{n=1}^{\infty} \frac{1}{n^3+1}$

$$\frac{1}{n^3+1} < \frac{1}{n^3} \quad \text{FOR } n = 1, 2, 3, \ldots$$

THIS SERIES $\boxed{\text{CONVERGES}}$ BY DIRECT COMPARISON WITH $\sum_{n=1}^{\infty} \frac{1}{n^3}$, THE CONVERGENT $p=3$ SERIES.

2. (2.5 points) Show that the series converges and find its sum.

$$\sum_{n=2}^{\infty} \frac{4}{3^{n+1}} = \frac{4}{3} \sum_{n=2}^{\infty} \left(\frac{1}{3}\right)^n$$

$$= \frac{4}{3}\left[\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n - 1 - \frac{1}{3}\right]$$

$$= \frac{4}{3}\left[\frac{1}{1-\frac{1}{3}} - 1 - \frac{1}{3}\right]$$

$$= \frac{4}{3}\left[\frac{3}{2} - 1 - \frac{1}{3}\right] = \boxed{\frac{2}{9}}$$