

Math 132 - Quiz 11

November 30, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due December 5.

1. (1 point) What is the center of the power series $\sum_{n=1}^{\infty} \frac{(3x+1)^n}{n^3}$?

$$3x+1 = 3\left(x + \frac{1}{3}\right) = 3\left(x - \left(-\frac{1}{3}\right)\right)$$

$$\text{Center} = \boxed{-\frac{1}{3}}$$

2. (3 points) Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$.

RATIO TEST :

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{x^{2n+2}}{(2n+2)!} \cdot \frac{(2n)!}{x^{2n}} \right| &= \lim_{n \rightarrow \infty} \frac{x^2}{(2n+2)(2n+1)} \\ &= x^2 \lim_{n \rightarrow \infty} \frac{1}{(2n+2)(2n+1)} = x^2 \cdot 0 \\ &= 0 \end{aligned}$$

THE RADIUS OF CONVERGENCE IS ∞ .

THE INTERVAL OF CONVERGENCE IS $(-\infty, \infty)$.

Turn over.

3. (6 points) Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-4)^n}{n 9^n}$.

RATIO TEST :

$$\lim_{n \rightarrow \infty} \left| \frac{(x-4)^{n+1}}{(n+1) 9^{n+1}} \cdot \frac{n 9^n}{(x-4)^n} \right|$$

$$= \lim_{n \rightarrow \infty} \frac{n |x-4|}{9(n+1)} = \frac{|x-4|}{9} \lim_{n \rightarrow \infty} \frac{n}{n+1}$$

$$= \frac{|x-4|}{9} \lim_{n \rightarrow \infty} \frac{1}{1+\frac{1}{n}} = \frac{|x-4|}{9}$$

$$\frac{|x-4|}{9} < 1 \Rightarrow -9 < x-4 < 9$$
$$\Rightarrow -5 < x < 13$$

RADIUS OF
CONVERGENCE
 $R = 9$

ENDPTS:

$$x = 13 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \text{ALT. HARM SERIES}$$

(CONVERGES)

$$x = -5 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot (-1)^n}{n} = \text{NEG. HARM SERIES}$$

(DIVERGES)

INTERVAL OF CONVERGENCE
IS $(-5, 13]$