

Math 132 - Quiz 3

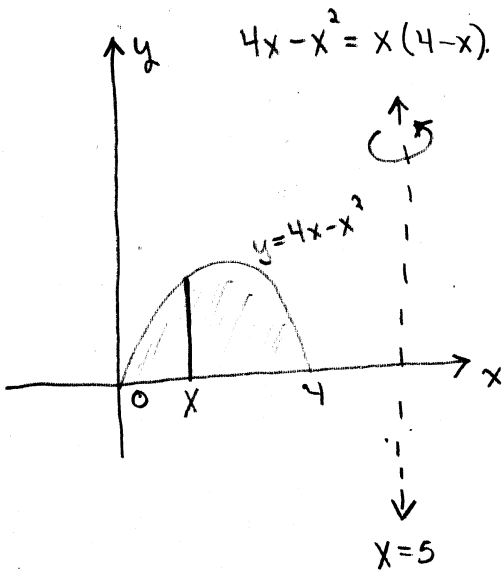
September 7, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due September 12.

1. (3 points) The 1st-quadrant region between the graphs $y = 4x - x^2$ and $y = 0$ is rotated about the line $x = 5$ to form a solid of revolution. Find the volume of the solid.



SHELLS...

$$\text{VOLUME} = 2\pi \int_0^4 (5-x)(4x-x^2) dx$$

$$= 2\pi \int_0^4 (20x - 9x^2 + x^3) dx$$

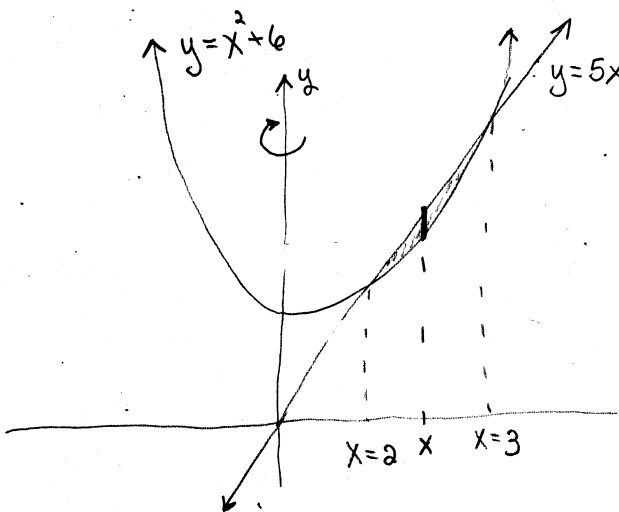
$$= 2\pi \left(10x^2 - 3x^3 + \frac{1}{4}x^4 \right) \Big|_0^4$$

$$= 2\pi (160 - 192 + 64) = \boxed{64\pi}$$

2. (3 points) The bounded region between the graphs of $y = x^2 + 6$ and $y = 5x$ is rotated about the y -axis to form a solid of revolution. Find the volume of the solid.

$$x^2 + 6 = 5x \Rightarrow x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$



SHELLS...

$$\text{VOLUME} = 2\pi \int_2^3 x(5x - x^2 - 6) dx$$

$$= 2\pi \int_2^3 (5x^2 - x^3 - 6x) dx$$

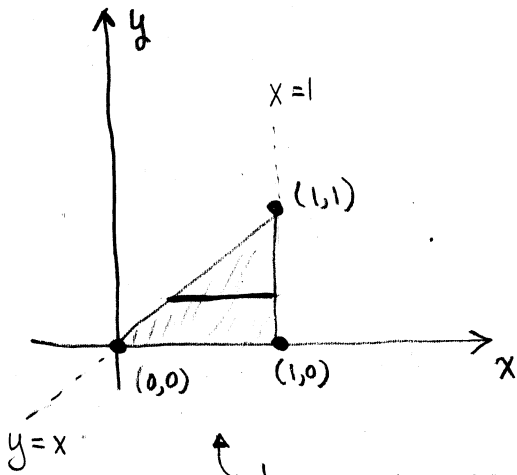
$$= 2\pi \left(\frac{5}{3}x^3 - \frac{1}{4}x^4 - 3x^2 \right) \Big|_2^3$$

$$= 2\pi \left(-\frac{9}{4} + \frac{8}{3} \right)$$

$$= \boxed{\frac{5\pi}{6}}$$

Turn over.

3. (2 points) The base of a solid is the triangle in the xy -plane with vertices $(0,0)$, $(1,0)$, and $(1,1)$. Cross sections (slices) perpendicular to the y -axis are squares. Find the volume of the solid.

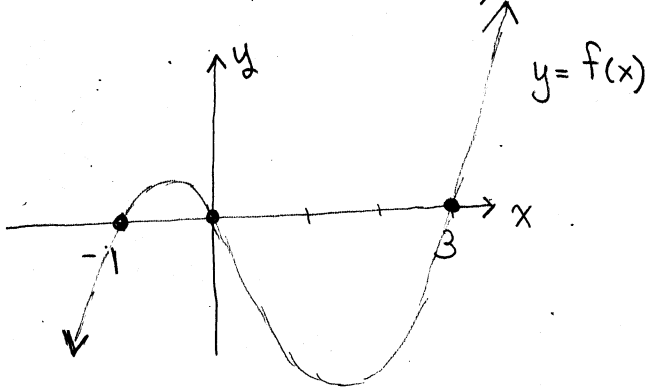


↑ LENGTH OF STRIP
AT y IS $1-y$.
∴ AREA OF CROSS SECTION
IS $(1-y)^2$

$$\begin{aligned} \text{Volume} &= \int_0^1 (1-y)^2 dy \\ &= \int_0^1 (1-2y+y^2) dy \\ &= y - y^2 + \frac{1}{3}y^3 \Big|_0^1 \\ &= \boxed{\frac{1}{3}} \end{aligned}$$

4. (2 points) Consider the portion of the graph of $f(x) = x^3 - 2x^2 - 3x$ that lies in the 4th quadrant. Set up the definite integral that gives the length of that portion. Use your calculator or computer to estimate the value of the integral.

$$\begin{aligned} f(x) &= x(x^2 - 2x - 3) \\ &= x(x-3)(x+1) \end{aligned}$$



ARC LENGTH =

$$\int_0^3 \sqrt{1 + (3x^2 - 4x - 3)^2} dx$$

$$\approx 12.695$$

$$f'(x) = 3x^2 - 4x - 3$$