Math 132 - Quiz 5

Name key

September 28, 2022

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 3.

1. (1 point) Evaluate the limit: $\lim_{x\to 0} \frac{\sinh x}{x}$ %

$\sinh(0) = \frac{e^0 - e^{-0}}{2} = \frac{1-1}{2} = 0$

$\cosh(0) = \frac{e^0 + e^{-0}}{2} = \frac{2}{2} = 1$

$= \lim_{x\to 0} \frac{\cosh x}{1}$ By l'Hopital's rule

$= \frac{\cosh(0)}{1} = \boxed{1}$

2. (1 point) Determine $f'(x)$ if $f(x) = \ln(\sinh x)$.

$f'(x) = \frac{1}{\sinh(x)} \cdot \frac{d}{dx}\sinh(x) = \frac{\cosh x}{\sinh x} = \boxed{\coth x}$

3. (1 point) Use the quotient rule to prove $\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$.

$\frac{d}{dx}\frac{1}{\cosh x} = \frac{(\cosh x)(0) - (1)(\sinh x)}{\cosh^2 x} = \frac{-\sinh x}{\cosh^2 x} = -\frac{1}{\cosh x}\cdot\frac{\sinh x}{\cosh x}$

$= -\operatorname{sech} x \tanh x$

4. (1 point) Evaluate the indefinite integral: $\int x \sinh x\, dx$

signs	u	dv/dx
+	x	$\sinh x$
−	1	$\cosh x$
+	0	$\sinh x$

$= \boxed{x\cosh x - \sinh x + C}$

Turn over.

THAT X SHOULD NOT HAVE BEEN THERE!

5. (2 points) Evaluate the indefinite integral: $\int x \arcsin x \, dx$ $= \frac{1}{2}x^2 \sin^{-1} x - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

$u = \sin^{-1} x \qquad du = \frac{1}{\sqrt{1-x^2}}\,dx$

$dv = x\,dx \qquad v = \frac{1}{2}x^2$

TRIG SUBSTITUTION!

WHOOPS! SORRY.

WE'LL GET TO TRIG. SUBS. VERY SOON...

6. (4 points) Use two different methods to evaluate $\int \sin(x)\cos(4x)\,dx$.

(a) Use a product-to-sum trig identity.

$$\int \sin x \cos 4x \, dx = \frac{1}{2}\int [\sin(5x) + \sin(-3x)]\,dx$$

$$= -\frac{1}{10}\cos(5x) + \frac{1}{6}\cos(3x) + C$$

$\cos(-3x) = \cos(3x)$

(b) Use integration by parts (twice).

signs	u	$\frac{dv}{dx}$
+	$\cos 4x$	$\sin x$
−	$-4\sin 4x$	$-\cos x$
+	$-16\cos 4x$	$-\sin x$

$$\int \sin x \cos 4x \, dx$$

$$= -\cos x \cos 4x - 4 \sin x \sin 4x + \int 16 \sin x \cos 4x \, dx$$

SUBTRACT FROM BOTH SIDES

$$\int \sin x \cos 4x \, dx = \frac{1}{15}\cos x \cos 4x + \frac{4}{15}\sin x \sin 4x + C$$