Math 132 - Quiz 6

October 5, 2022

Name key

Score ____________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 10.

1. (2 points) Evaluate the indefinite integral: $\displaystyle\int \tan^4 3x \sec^2 3x \, dx$

$u = \tan 3x$

$du = 3 \sec^2 3x \, dx$

$\frac{1}{3} du = \sec^2 3x \, dx$

$$\frac{1}{3}\int u^4 \, du = \frac{1}{15} u^5 + C$$

$$= \boxed{\frac{1}{15} \tan^5 (3x) + C}$$

2. (2 points) Evaluate the indefinite integral: $\displaystyle\int \sin^4 x \cos^5 x \, dx$

$$\int \sin^4 x \left(1 - \sin^2 x\right)^2 \cos x \, dx = \int u^4 (1-u^2)^2 \, du$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int \left(u^4 - 2u^6 + u^8\right) du$$

$$= \frac{1}{5} u^5 - \frac{2}{7} u^7 + \frac{1}{9} u^9 + C$$

$$= \boxed{\frac{1}{5} \sin^5 x - \frac{2}{7} \sin^7 x + \frac{1}{9} \sin^9 x + C}$$

Turn over.

Use $\sin^2 x = \dfrac{1-\cos 2x}{2}$

3. (2 points) Evaluate the indefinite integral: $\displaystyle\int x^2 \sin^2 x\, dx$

$$\frac{1}{2}\int \left(x^2 - x^2\cos 2x\right) dx$$

$$\int x^2 \cos 2x\, dx$$

$+$	x^2	$\cos 2x$
$-$	$2x$	$\frac{1}{2}\sin 2x$
$+$	2	$-\frac{1}{4}\cos 2x$
$-$	0	$-\frac{1}{8}\sin 2x$

$$= \frac{1}{2}\left[\frac{1}{3}x^3 - \left(\frac{1}{2}x^2\sin 2x + \frac{x}{2}\cos 2x - \frac{1}{4}\sin 2x\right)\right] + C$$

$$= \frac{1}{6}x^3 - \frac{1}{4}x^2\sin 2x - \frac{x}{4}\cos 2x + \frac{1}{8}\sin 2x + C$$

4. (2 points) Evaluate the definite integral: $\displaystyle\int_0^{\pi/2} \sin^5 x\, dx$

$$\int_0^{\pi/2} \sin^5 x\, dx = \frac{1}{16}\int_0^{\pi/2} \left(\sin 5x - 5\sin 3x + 10\sin x\right) dx$$

$$= \frac{1}{16}\left[-\frac{1}{5}\cos 5x + \frac{5}{3}\cos 3x - 10\cos x\right]_0^{\pi/2}$$

$$= \frac{1}{16}\left[(0+0-0) - \left(-\frac{1}{5} + \frac{5}{3} - 10\right)\right] = \frac{1}{16}\cdot\frac{128}{15} = \frac{8}{15}$$

5. (2 points) Evaluate the indefinite integral: $\displaystyle\int \tan^5 7x \sec 7x\, dx$

$$\int \tan^4 7x \tan 7x \sec 7x\, dx$$

$$= \int \left(\sec^2 7x - 1\right)^2 \tan 7x \cdot \sec 7x\, dx = \frac{1}{7}\int \left(u^2-1\right)^2 du = \frac{1}{7}\int \left(u^4 - 2u^2 + 1\right) du$$

$u = \sec 7x$

$du = 7\sec 7x \tan 7x\, dx$

$$= \frac{1}{7}\left(\frac{1}{5}u^5 - \frac{2}{3}u^3 + u\right) + C$$

$$= \frac{1}{35}\sec^5 7x - \frac{2}{21}\sec^3 7x + \frac{1}{7}\sec 7x + C$$