Math 132 - Quiz 7

October 12, 2022

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 17.

1. (3 points) Evaluate the indefinite integral: $\int \sqrt{5+2x^2}\,dx$

$\sqrt{2}\,x = \sqrt{5}\tan\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\sqrt{2}\,dx = \sqrt{5}\sec^2\theta\,d\theta$

$$\int \sqrt{5+5\tan^2\theta}\left(\frac{\sqrt{5}}{\sqrt{2}}\right)\sec^2\theta\,d\theta$$

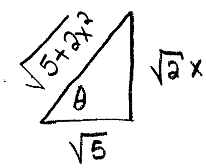
$$= \frac{\sqrt{5}}{\sqrt{2}}\int\sqrt{5\sec^2\theta}\,\sec^2\theta\,d\theta$$

$$= \frac{5}{\sqrt{2}}\int|\sec\theta|\sec^2\theta\,d\theta$$

$$= \frac{5}{\sqrt{2}}\int\sec^3\theta\,d\theta$$

SEE ATTACHED SHEET

$$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta| + C$$



$$\frac{1}{2}\,\frac{\sqrt{5+2x^2}}{\sqrt{5}}\,\frac{\sqrt{2}\,x}{\sqrt{5}} + \frac{1}{2}\ln\left|\frac{\sqrt{5+2x^2}}{\sqrt{5}} + \frac{\sqrt{2}\,x}{\sqrt{5}}\right| + C$$

2. (2 points) Evaluate the indefinite integral: $\int \frac{x\,dx}{\sqrt{4-x^2}}$

$u = 4-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$$-\frac{1}{2}\int u^{-1/2}\,du = -u^{1/2} + C$$

$$= \frac{-1}{\sqrt{4-x^2}} + C$$

Turn over.

3. (3 points) Evaluate the indefinite integral: $\int \sqrt{2x - x^2}\, dx$

(Hint: Complete the square.)

$$2x - x^2 = 1 - (x-1)^2$$

$$\int \sqrt{1-(x-1)^2}\, dx$$

$x - 1 = \sin\theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

$dx = \cos\theta\, d\theta$

$$\int \sqrt{1 - \sin^2\theta}\, \cos\theta\, d\theta$$

$$= \int |\cos\theta| \cos\theta\, d\theta$$

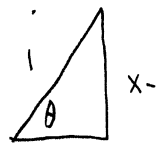
$$= \int \cos^2\theta\, d\theta$$

$$= \int \frac{1}{2} + \frac{1}{2}\cos 2\theta\, d\theta$$

Use $\sin 2\theta = 2 \sin\theta \cos\theta$

$$= \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C$$

$$= \frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta + C$$



$\sqrt{1-(x-1)^2}$

$= \sqrt{2x - x^2}$

$$\frac{1}{2}\sin^{-1}(x-1) + \frac{1}{2}(x-1)\sqrt{2x-x^2}$$

4. (2 points) Evaluate the indefinite integral: $\int \frac{8x-1}{x^2 - x - 2}\, dx$

$$\frac{8x-1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

Cover up ...

$A = \frac{15}{3} = 5$

$B = \frac{-9}{-3} = 3$

$$= \int \frac{5}{x-2}\, dx + \int \frac{3}{x+1}\, dx$$

$$= 5 \ln|x-2| + 3 \ln|x+1| + C$$

$\int \sec^3\theta \, d\theta$ WE DID THIS IN CLASS. USE INTEGRATION BY PARTS. IT IS ALSO IN THE LECTURE NOTES.

$u = \sec\theta$ $\qquad du = \sec\theta \tan\theta \, d\theta$

$dv = \sec^2\theta \, d\theta$ $\qquad v = \tan\theta$

$$\int \sec^3\theta \, d\theta = \sec\theta\tan\theta - \int \tan^2\theta \sec\theta \, d\theta$$

$$= \sec\theta\tan\theta - \int (\sec^2\theta - 1)\sec\theta \, d\theta$$

$$= \sec\theta\tan\theta \underbrace{- \int \sec^3\theta \, d\theta}_{\text{ADD TO BOTH SIDES}} + \int \sec\theta \, d\theta$$

$$2\int \sec^3\theta \, d\theta = \sec\theta\tan\theta + \ln|\sec\theta + \tan\theta| + C$$

$$\int \sec^3\theta \, d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$