Math 132 - Quiz 8

October 26, 2022

Name Key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 31.

1. (2 points) Use the trapezoid rule over 5 subintervals to approximate $\int_1^3 \frac{e^x}{x}\,dx$.

$h = \frac{3-1}{5} = \frac{2}{5} = 0.4$

$x_0 = 1$

$x_1 = 1.4$

$x_2 = 1.8$

$x_3 = 2.2$

$x_4 = 2.6$

$x_5 = 3.0$

TRAP RULE GIVES

$$\int_1^3 \frac{e^x}{x}\,dx \approx \frac{0.4}{2}\left[e + \frac{2e^{1.4}}{1.4} + \frac{2e^{1.8}}{1.8} + \frac{2e^{2.2}}{2.2} + \frac{2e^{2.6}}{2.6} + \frac{e^3}{3}\right]$$

$$\approx \boxed{8.0979}$$

2. (2 points) Use Simpson's rule over 4 subintervals to approximate $\int_0^4 (x^3 + 5x)\,dx$. Compare your answer to the exact value of the integral.

$h = \frac{4-0}{4} = 1$

$x_0 = 0$

$x_1 = 1$

$x_2 = 2$

$x_3 = 3$

$x_4 = 4$

$f(x) = x^3 + 5x$

SIMP'S RULE GIVES

$$\int_0^4 (x^3+5x)\,dx \approx \frac{1}{3}\left[f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)\right]$$

$$= \frac{1}{3}(0 + 24 + 36 + 168 + 84)$$

$$= \boxed{104}$$

EXACT VALUE =

$$\left.\frac{1}{4}x^4 + \frac{5}{2}x^2\right|_0^4 = \boxed{104}$$

SAME! IN FACT, SIMPSON'S RULE IS EXACT FOR CUBIC POLYNOMIALS.

Turn over.

3. (3 points) Suppose s is a positive constant. Evaluate the improper integral $\int_0^\infty xe^{-sx}\,dx$.

$$\int xe^{-sx}\,dx = \left(-\frac{x}{s} - \frac{1}{s^2}\right)e^{-sx} + C$$

+	x	e^{-sx}
−	1	$-\frac{1}{s}e^{-sx}$
+	0	$\frac{1}{s^2}e^{-sx}$

$$\int_0^\infty xe^{-sx}\,dx = \lim_{t\to\infty}\int_0^t xe^{-sx}\,dx = \lim_{t\to\infty}\left(\frac{x}{s}+\frac{1}{s^2}\right)e^{-sx}\Big|_t^0$$

$$= \frac{1}{s^2} - \lim_{t\to\infty}\left(\underbrace{\frac{t}{se^{st}}} + \cancel{\frac{1}{s^2e^{st}}}\right)$$

$1/\infty$ FORM. THIS LIMIT IS ZERO, ASSUMING $s>0$.

$$\lim_{t\to\infty}\frac{t}{se^{st}} \overset{\infty/\infty}{\underset{\text{L.H. RULE}}{=}} \underbrace{\lim_{t\to\infty}\frac{1}{s^2e^{st}} = 0}_{\text{ASSUMING } s>0}$$

$$\boxed{\int_0^\infty xe^{-sx}\,dx = \frac{1}{s^2}}$$

4. (2 points) Evaluate $\int_{-1}^1 \frac{1}{x^2}\,dx$. $= \int_{-1}^0 \frac{1}{x^2}dx + \int_0^1 \frac{1}{x^2}dx$

$$\lim_{t\to0^+}\int_t^1 x^{-2}\,dx = \lim_{t\to0^+} -\frac{1}{x}\Big|_t^1$$

$$= \lim_{t\to0^+}\left[-1+\frac{1}{t}\right] = \infty$$

DIVERGES.

$\int_{-1}^1 \frac{1}{x^2}dx$ DIVERGES.

5. (1 point) Explain why $\int_0^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$ is improper.

IT IS IMPROPER FOR BOTH OF THE REASONS WE STUDIED:

1. THE INTEGRATION INTERVAL IS UNBOUNDED, $(0,\infty)$; AND
2. $\frac{e^{-\sqrt{x}}}{\sqrt{x}}$ HAS AN INFINITE DISCONTINUITY AT $x=0$.