Math 132 - Quiz 9

November 2, 2022

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due November 7.

1. (1 point) Find the first five terms of the sequence with $a_1 = 2$ and $a_n = \frac{1}{2}(a_{n-1} + 2n)$ for $n = 2, 3, 4, \ldots$.

$a_1 = 2$

$a_2 = \frac{1}{2}(2+4) = 3$

$a_3 = \frac{1}{2}(3+6) = \frac{9}{2}$

$a_4 = \frac{1}{2}\left(\frac{9}{2}+8\right) = \frac{25}{4}$

$a_5 = \frac{1}{2}\left(\frac{25}{4}+10\right) = \frac{65}{8}$

$$\boxed{\{a_n\} = \left\{2, 3, \frac{9}{2}, \frac{25}{4}, \frac{65}{8}, \ldots\right\}}$$

2. (2 points) Find a formula for a_n, where $a_1 = -4$ and $a_{n+1} - a_n = 13$ for $n = 1, 2, 3, \ldots$.

$a_{n+1} = a_n + 13$

$a_1 = -4$, $a_2 = -4 + 13$, $a_3 = -4 + 2(13)$,

$a_4 = -4 + 3(13)$, ...

$$\boxed{a_n = -4 + 13(n-1) \quad \text{or} \quad a_n = 13n - 17, \quad n = 1, 2, 3, \ldots}$$

3. (1.5 points) Find the limit of the sequence $\{a_n\}_{n=1}^{\infty}$, where $a_n = \dfrac{\sqrt{n}}{\sqrt{n+5}}$.

Let $f(x) = \dfrac{\sqrt{x}}{\sqrt{x+5}}$, $x \geq 0$

$$\lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x+5}} = \lim_{x\to\infty} \sqrt{\frac{x}{x+5}} = \lim_{x\to\infty} \sqrt{\frac{1}{1+\frac{5}{x}}} = 1$$

$\therefore$ $\boxed{a_n \to 1 \text{ as } n \to \infty}$

Turn over.

4. (3 points) Find the limit of each sequence or show that it diverges.

(a) $\left\{ \frac{\ln(n^2)}{\ln(2n)} \right\}_{n=1}^{\infty}$

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$$\lim_{n\to\infty} \frac{\ln(n^2)}{\ln(2n)} = \lim_{n\to\infty} \frac{2\ln(n)}{\ln(2)+\ln(n)} \cdot \frac{\frac{1}{\ln(n)}}{\frac{1}{\ln(n)}} = \lim_{n\to\infty} \frac{2}{\frac{\ln(2)}{\ln(n)}+1}$$

$$= \frac{2}{0+1} = \boxed{2}$$

(b) $\left\{ \frac{2^n+3^n}{4^n} \right\}_{n=0}^{\infty}$

$$\lim_{n\to\infty} \frac{2^n+3^n}{4^n} = \lim_{n\to\infty} \left(\frac{2^n}{4^n} + \frac{3^n}{4^n} \right)$$

$$= \lim_{n\to\infty} \left(\frac{2^n}{4^n} \right) + \lim_{n\to\infty} \left(\frac{3^n}{4^n} \right) = \lim_{n\to\infty} \left(\frac{1}{2} \right)^n + \lim_{n\to\infty} \left(\frac{3}{4} \right)^n$$

$$= 0 + 0 = \boxed{0}$$

5. (2.5 points) Find a formula for the nth partial sum of the series $\sum_{n=1}^{\infty} \frac{2}{n^2+2n}$. Does the series converge or diverge?

$$\frac{2}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$$

Cover up...

$A = 1, \ B = -1$

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2} \right)$$

$$= \underbrace{\left(\frac{1}{1} - \cancel{\frac{1}{3}} \right)}_{n=1} + \underbrace{\left(\frac{1}{2} - \cancel{\frac{1}{4}} \right)}_{n=2} + \underbrace{\left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}} \right)}_{n=3} + \underbrace{\left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}} \right)}_{n=4}$$

$$+ \underbrace{\left(\cancel{\frac{1}{5}} - \frac{1}{7} \right)}_{n=5} + \underbrace{\left(\frac{1}{6} - \frac{1}{8} \right)}_{n=6} + \cdots$$

$$S_n = 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} = \frac{3}{2} - \left(\frac{1}{n+1} + \frac{1}{n+2} \right)$$

$S_n \to \frac{3}{2}$ AS $n \to \infty$. Series converges. Sum $= 3/2$