Math 132 - Quiz 1

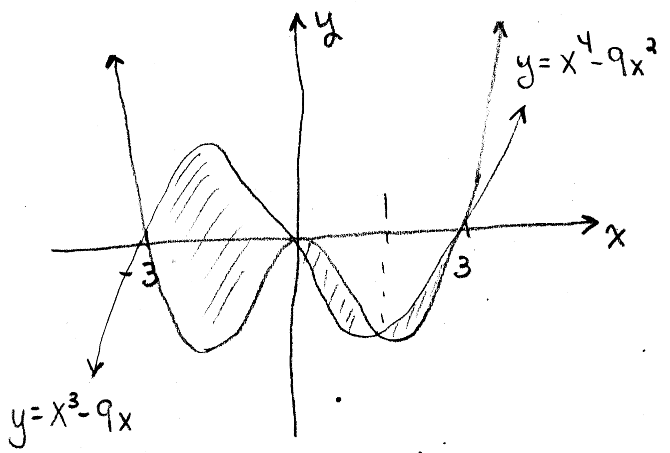
January 23, 2020

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due no later than 3:15pm on January 28.

1. (5 points) Find the total combined area of the bounded regions between the graphs of $y = x^4 - 9x^2$ and $y = x^3 - 9x$.



$$x^4 - 9x^2 = x^2(x^2-9) = x^2(x+3)(x-3)$$

$$x^3 - 9x = x(x^2-9) = x(x+3)(x-3)$$

INTERSECTION PTS...

$$x^4 - 9x^2 = x^3 - 9x$$

$$0 = x^4 - x^3 - 9x^2 + 9x = x(x^3 - x^2 - 9x + 9)$$

$$= x(x^2(x-1) - 9(x-1))$$

$$= x(x^2-9)(x-1)$$

$$\Rightarrow x = 0, 3, -3, 1$$

$$\text{AREA} = \int_{-3}^{0} (x^3-9x) - (x^4-9x^2)\, dx + \int_0^1 (x^4-9x^2) - (x^3-9x)\, dx + \int_1^3 (x^3-9x) - (x^4-9x^2)\, dx$$

$$= \left(\frac{1}{4}x^4 - \frac{9}{2}x^2 - \frac{1}{5}x^5 + 3x^3\right)\Bigg|_{-3}^{0} + \left(\frac{1}{5}x^5 - 3x^3 - \frac{1}{4}x^4 + \frac{9}{2}x^2\right)\Bigg|_0^1 + \left(\frac{1}{4}x^4 - \frac{9}{2}x^2 - \frac{1}{5}x^5 + 3x^3\right)\Bigg|_1^3$$

$$= \left[0 - \left(\frac{81}{4} - \frac{81}{2} + \frac{243}{5} - 81\right)\right] + \left[\left(\frac{1}{5} - 3 - \frac{1}{4} + \frac{9}{2}\right) - 0\right]$$

Turn over.

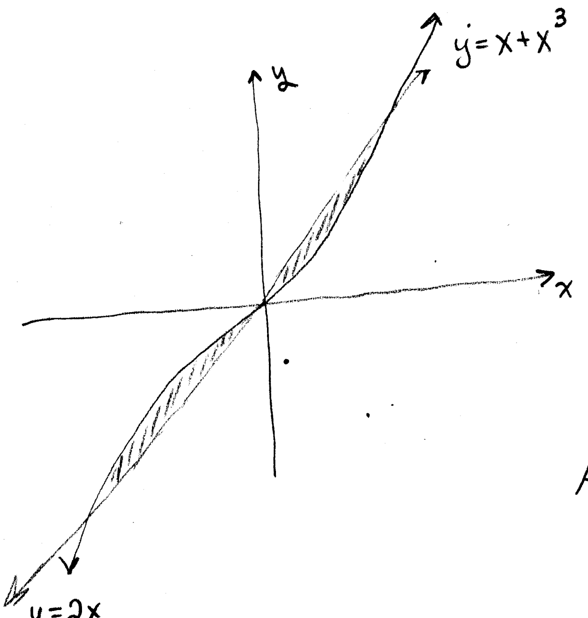
$$+ \left[\left(\frac{81}{4} - \frac{81}{2} - \frac{243}{5} + 81\right) - \left(\frac{1}{4} - \frac{9}{2} - \frac{1}{5} + 3\right)\right] = \frac{1053}{20} + \frac{29}{20} + \frac{68}{5}$$

$$= \boxed{\frac{677}{10} \text{ UNITS}^2}$$

2. (5 points) Find the total combined area of the bounded regions between the graphs of $y + y^3 = x$ and $2y = x$.

INSTEAD LET'S RENAME VARIABLES...

$$y = x + x^3$$
$$y = 2x$$



$$x + x^3 = 2x \Rightarrow x^3 - x = 0$$
$$x(x^2-1) = 0$$
$$x = 0, 1, -1$$

$$\text{AREA} = \int_{-1}^{0} (x + x^3 - 2x)\,dx + \int_0^1 (2x - x - x^3)\,dx$$

$$= \int_{-1}^{0} (x^3 - x)\,dx + \int_0^1 (x - x^3)\,dx$$

$$= \left(\frac{1}{4}x^4 - \frac{1}{2}x^2\right)\Big|_{-1}^{0} + \left(\frac{1}{2}x^2 - \frac{1}{4}x^4\right)\Big|_0^1$$

$$= \left[0 - \left(\frac{1}{4} - \frac{1}{2}\right)\right] + \left[\left(\frac{1}{2} - \frac{1}{4}\right) - 0\right]$$

$$= \frac{1}{4} + \frac{1}{4} = \boxed{\frac{1}{2} \text{ UNITS}^2}$$