Math 132 - Quiz 2 (IC)

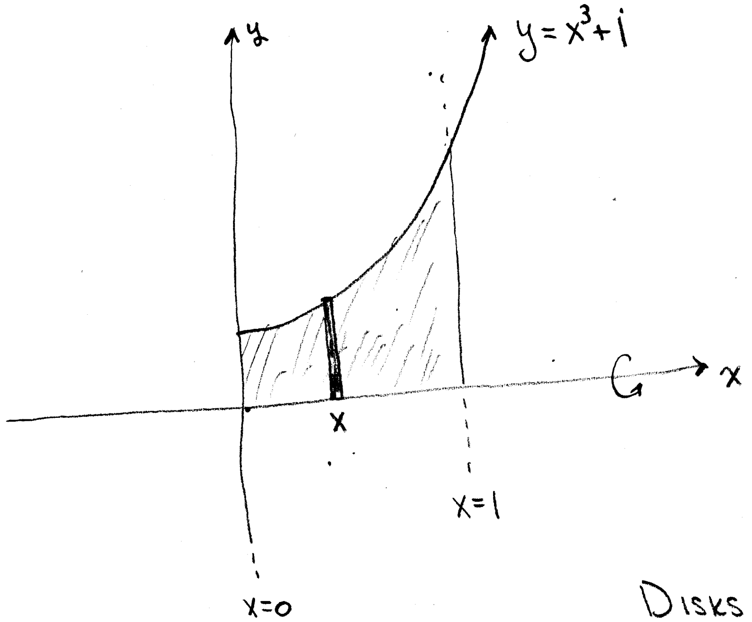
January 30, 2020

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) The 1st quadrant region bounded by the graphs of $y = x^3 + 1$, $x = 0$, $x = 1$, and $y = 0$ is rotated about the x-axis to form a solid. Sketch the region. Then find the volume of the solid.



Disks ....

$$\text{Volume} = \pi \int_0^1 (x^3+1)^2 \, dx$$

$$= \pi \int_0^1 (x^6 + 2x^3 + 1) \, dx$$

$$= \pi \left(\frac{1}{7}x^7 + \frac{1}{2}x^4 + x \right) \Big|_0^1$$

$$= \pi \left(\frac{1}{7} + \frac{1}{2} + 1 \right)$$

$$= \frac{23\pi}{14}$$

Math 132 - Quiz 2 (TH)

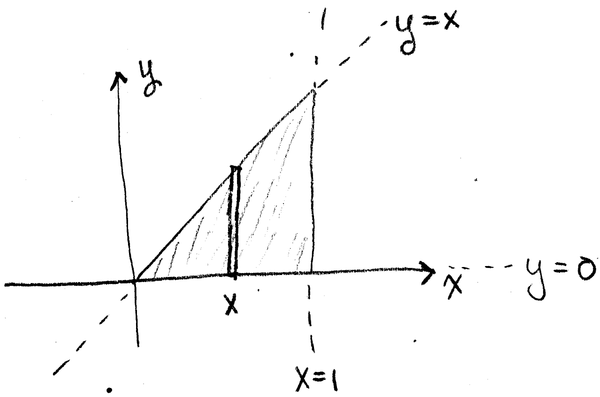
January 30, 2020

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due no later than 3:15pm on February 4.

1. (2 points) The base of a solid is the 1st quadrant region bounded by the graphs of $y = x$, $x = 1$, and $y = 0$. The cross sections perpendicular to the x-axis are squares. Find the volume of the solid.



$$\int_0^1 (\text{AREA OF CROSS SEC AT } x)\,dx$$

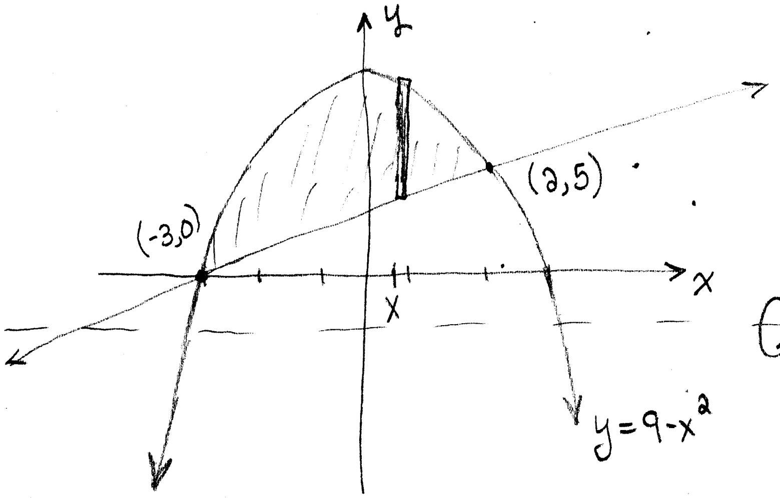
$$= \int_0^1 \left(\begin{matrix}\text{LENGTH OF STRIP}\\ \text{AT } x\end{matrix}\right)^2 dx$$

$$= \int_0^1 x^2\,dx$$

$$= \frac{1}{3}x^3\Big|_0^1 = \frac{1}{3}$$

Turn over.

2. (5 points) The bounded region between the graphs of $f(x) = 9 - x^2$ and $g(x) = x + 3$ is rotated about the line $y = -1$ to form a solid. Find the volume of the solid.



$G \to y=-1$

$$9-x^2 = x+3$$
$$x^2+x-6=0$$
$$(x+3)(x-2)=0$$
$$x=-3 \quad x=2$$

WASHERS...

$$\text{Volume} = \pi\int_{-3}^{2}\left[(9-x^2+1)^2 - (x+3+1)^2\right] dx$$

$$= \pi\int_{-3}^{2} (10-x^2)^2 - (x+4)^2 \, dx$$

$$= \pi\int_{-3}^{2} (x^4-20x^2+100) - (x^2+8x+16) \, dx$$

$$= \pi\int_{-3}^{2} (x^4-21x^2-8x+84) \, dx$$

$$= \pi\left[\frac{1}{5}x^5 - 7x^3 - 4x^2 + 84x\right]_{-3}^{2}$$

$$= \pi\left[\left(\frac{32}{5} - 56 - 16 + 168\right) - \left(\frac{-243}{5} + 189 - 36 - 252\right)\right]$$

$$= \boxed{250\pi}$$