Math 132 - Quiz 4

April 2, 2020

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. You must work individually on this quiz. This quiz is due no later than April 7.

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1. (2 points) Use the trapezoid rule over five subintervals to approximate $\int_0^1 e^{-x^2}\,dx$. Then use your calculator or computer to approximate the integral and compare the values.

$f(x) = e^{-x^2}$; $h = \frac{1}{5}$; $x_0 = 0$, $x_1 = \frac{1}{5}$, $x_2 = \frac{2}{5}$, $x_3 = \frac{3}{5}$, $x_4 = \frac{4}{5}$, $x_5 = 1$

$$T = \frac{1}{10}\left(f(0) + 2f\left(\tfrac{1}{5}\right) + 2f\left(\tfrac{2}{5}\right) + 2f\left(\tfrac{3}{5}\right) + 2f\left(\tfrac{4}{5}\right) + f(1)\right)$$

$$\approx \boxed{0.744368}$$

VALUE FROM SAGEMATH IS ≈ 0.746824

2. (2 points) Rewrite as a limit and evaluate: $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$

$$\lim_{t\to 1^-} \int_0^t \frac{1}{\sqrt{1-x^2}}\,dx = \lim_{t\to 1^-} \sin^{-1} x \Big|_0^t$$

$$= \lim_{t\to 1^-} \left(\sin^{-1} t - \sin^{-1} 0\right)$$

$$= \lim_{t\to 1^-} \sin^{-1} t = \boxed{\frac{\pi}{2}}$$

3. (2 points) Rewrite as a limit and evaluate: $\displaystyle\int_1^\infty \frac{\ln x}{x}\,dx$

I'LL SUBSTITUTE FIRST.

$u = \ln x$

$du = \frac{1}{x}dx$

$x = 1 \Rightarrow u = 0$

$x \to \infty \Rightarrow u \to \infty$

$$\int_0^\infty u\,du = \lim_{t\to\infty} \int_0^t u\,du = \lim_{t\to\infty} \left(\frac{1}{2}u^2 \Big|_0^t \right)$$

$$= \lim_{t\to\infty} \left(\frac{1}{2}t^2 \right) = \infty \quad \boxed{\text{DIVERGES}}$$

4. (2 points) Write the first five terms of the sequence whose nth term is $a_n = \frac{n^3}{2^n}$. Then determine whether the sequence converges or diverges. If it converges, find the limit.

TERMS ARE $\frac{1}{2}, \frac{8}{4}, \frac{27}{8}, \frac{64}{16}, \frac{125}{32}, \ldots$

$$\boxed{\lim_{n\to\infty} \frac{n^3}{2^n} = 0}$$

LET $f(x) = \frac{x^3}{2^x}$. NOTICE THAT $f(n) = \frac{n^3}{2^n}$.

$$\underbrace{\lim_{x\to\infty} \frac{x^3}{2^x} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{3x^2}{2^x \ln 2} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{6x}{2^x (\ln 2)^2} \overset{\frac{\infty}{\infty}}{=}}_{\text{L'HOPITAL'S RULE THREE TIMES!}} \lim_{x\to\infty} \frac{6}{2^x (\ln 2)^3} \overset{\frac{6}{\infty}}{=} 0$$

5. (2 points) Determine whether each sequence converges or diverges. If it converges, find the limit.

(a) $a_n = \dfrac{\ln(n^2)}{\ln(2n)}$

LET $f(x) = \frac{\ln x^2}{\ln 2x}$

$$\boxed{\lim_{n\to\infty} a_n = 2}$$

$$\underbrace{\lim_{x\to\infty} \frac{\ln x^2}{\ln 2x} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{2x/x^2}{2/2x}}_{\text{L'HOPITAL}} = \lim_{x\to\infty} \frac{4x^2}{2x^2} = 2$$

(b) $a_n = \ln\left(\dfrac{n+2}{n^2}\right)$

SINCE

$\lim_{n\to\infty} \frac{n+2}{n^2} = 0$ AND $\ln x \to -\infty$ AS $x \to 0^+$,

IT FOLLOWS THAT $\lim_{n\to\infty} a_n = -\infty$

SEQUENCE DIVERGES