Math 132 - Quiz 5

April 9, 2020

Name key

Score ____________

Show all work to receive full credit. Supply explanations when necessary. You must work individually on this quiz. This quiz is due no later than April 14.

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1. (2 points) Consider the infinite series $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$.

   (a) After rewriting $\ln\left(\frac{n}{n+1}\right) = \ln n - \ln(n+1)$, you should see that this series is a telescoping series. Find a formula for the nth partial sum, S_n.

$S_1 = \ln 1 - \ln 2$

$S_2 = (\ln 1 - \ln 2) + (\ln 2 - \ln 3)$
$= \ln 1 - \ln 3$

$S_3 = (\ln 1 - \ln 2) + (\ln 2 - \ln 3) + (\ln 3 - \ln 4)$
$= \ln 1 - \ln 4$

IN GENERAL, $S_n = \ln 1 - \ln(n+1) = -\ln(n+1)$

   (b) Determine whether the series converges or diverges. Explain your reasoning.

$$\underbrace{\lim_{n\to\infty} S_n = \lim_{n\to\infty} -\ln(n+1) = -\infty}$$

SINCE THE SEQUENCE OF PARTIAL SUM DIVERGES, THE SERIES DIVERGES.

2. (2 points) Consider the series $\sum_{k=1}^{\infty} \frac{\pi}{3^k}$.

   (a) This series converges. Explain in a phrase or sentence how we that it converges.

$$\sum_{k=1}^{\infty} \pi\left(\frac{1}{3}\right)^k$$

THE SERIES IS GEOMETRIC WITH $r = \frac{1}{3} < 1$.

   (b) Find the sum of the series.

$$\sum_{k=0}^{\infty} \pi\left(\frac{1}{3}\right)^k = \frac{\pi}{1 - 1/3} = \frac{3\pi}{2}$$

BUT THIS INCLUDES THE $k=0$ TERM, AND THE ORIGINAL DOES NOT.

$$\sum_{k=1}^{\infty} \pi\left(\frac{1}{3}\right)^k = \frac{3\pi}{2} - \pi\left(\frac{1}{3}\right)^0 = \frac{\pi}{2}$$

3. (2 points) For each problem below, consider the series $\sum_{n=1}^{\infty} a_n$. Apply the nth term test and describe the conclusion of the test.

(a) $a_n = e^{-2/n}$ As $n \to \infty$, $\frac{2}{n} \to 0$. THEREFORE $\lim_{n\to\infty} e^{-2/n} = e^0 = 1$

$\sum_{n=1}^{\infty} e^{-2/n}$ DIVERGES.

(b) $a_n = \frac{(\ln n)^2}{\sqrt{n}}$ LET $f(x) = \frac{(\ln x)^2}{\sqrt{x}}$

$$\lim_{x\to\infty} \underbrace{\frac{(\ln x)^2}{x^{1/2}} = \lim_{x\to\infty} \frac{(2\ln x)(\frac{1}{x})}{\frac{1}{2}x^{-1/2}}}_{\text{L'HOPITAL}} = \lim_{x\to\infty} \underbrace{\frac{4\ln x}{x^{1/2}} = \lim_{x\to\infty} \frac{4/x}{\frac{1}{2}x^{-1/2}}}_{\text{L'HOPITAL}} = \lim_{x\to\infty} \frac{8}{x^{1/2}} = 0$$

SINCE $\lim_{n\to\infty} a_n = 0$, THE TEST HAS NO CONCLUSION.

4. (2 points) Consider the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$. Does the integral test apply? Explain. If it applies, use it to determine convergence or divergence.

$f(x) = \frac{1}{x(\ln x)^2}$, $x \geq 2$

f IS POS. (PRETTY CLEARLY)

f IS CONT. (PRETTY CLEARLY)

f IS DECREASING. (MAKE X BIGGER, MAKES DENOM BIGGER, MAKES FRACTION SMALLER)

INTEGRAL TEST APPLIES...

$$\int_2^{\infty} \frac{1}{x(\ln x)^2}dx = \int_{\ln 2}^{\infty} u^{-2}du = \lim_{t\to\infty}\int_{\ln 2}^{t} u^{-2}du$$

$u = \ln x$

$du = \frac{1}{x}dx$

$$= \lim_{t\to\infty}\left(-\frac{1}{u}\Big|_{\ln 2}^{t}\right)$$

$$= \lim_{t\to\infty}\left(-\frac{1}{t} + \frac{1}{\ln 2}\right) = \frac{1}{\ln 2}$$

INTEGRAL CONVERGES $\Rightarrow$ SERIES CONVERGES.

5. (2 points) Use either direct or limit comparison to determine whether $\sum_{k=1}^{\infty} \frac{1}{5k^2 - 3k}$ converges or diverges.

FOR LARGE k, THE SERIES "LOOKS LIKE" $\sum \frac{1}{5k^2}$.

LET'S USE LIMIT COMP. WITH THE CONVERGENT P-SERIES $\sum \frac{1}{k^2}$

$$\lim_{k\to\infty} \frac{\frac{1}{5k^2-3k}}{\frac{1}{k^2}} = \lim_{k\to\infty} \frac{k^2}{5k^2-3k} \cdot \frac{1/k^2}{1/k^2}$$

$$= \lim_{k\to\infty} \frac{1}{5 - \frac{3}{k}} = \frac{1}{5}$$

$\sum_{k=1}^{\infty} \frac{1}{5k^2-3k}$ CONVERGES BY LIMIT COMPARISON.