Math 132 - Quiz 7
April 30, 2020

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary. You must work individually on this quiz. This quiz is due no later than May 5.

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1. (2 points) Let $f(x) = 1 + x + x^2$. Determine the 2nd Taylor polynomial for f at $x = 1$.

$f(x) = 1 + x + x^2, \quad f(1) = 3$

$f'(x) = 1 + 2x, \quad f'(1) = 3$

$f''(x) = 2, \quad f''(1) = 2$

$P_2(x) = 3 + 3(x-1) + \frac{2}{2}(x-1)^2.$

$P_2(x) = 3 + 3(x-1) + (x-1)^2$

2. (2 points) Use a computer algebra system to check your result in problem #1. (The SageMath syntax is `taylor(1+x+x^2,x,1,2)`.) Then expand your result (by doing the algebra). What do you notice?

$$3 + 3(x-1) + (x-1)^2 = 3 + 3x - 3 + x^2 - 2x + 1$$

$$= 1 + x + x^2$$

NOTICE THAT $P_2(x) = f(x)$

Follow-up: What do you think the 3rd Taylor polynomial for f at $x = 1$ would be?

SINCE $f'''(x) = 0$, $P_3(x) = P_2(x)$.

FOR THAT MATTER $P_n(x) = P_2(x)$ FOR ANY $n \geq 2$.

3. (3 points) Compute the 4th Maclaurin polynomial for $f(x) = e^{3x}$. Then use it to approximate $e^{0.75}$.

$f(x) = e^{3x}$, $f(0) = 1$

$f'(x) = 3e^{3x}$, $f'(0) = 3$

$f''(x) = 9e^{3x}$, $f''(0) = 9$

$f'''(x) = 27e^{3x}$, $f'''(0) = 27$

$f^{(4)}(x) = 81e^{3x}$, $f^{(4)}(0) = 81$

$$P_4(x) = 1 + 3x + \frac{9}{2}x^2 + \frac{27}{6}x^3 + \frac{81}{24}x^4$$

$$P_4(x) = 1 + 3x + \frac{9}{2}x^2 + \frac{9}{2}x^3 + \frac{27}{8}x^4$$

$$e^{0.75} \approx P_4(0.25) = 2.1147460 9375$$

4. (3 points) Find the Maclaurin series for $f(x) = e^{3x}$. Then use the ratio test to show that it converges for all x.

IT IS PRETTY OBVIOUS THAT $f^{(n)}(x) = 3^n e^{3x}$

SO THAT THE MAC. SERIES IS $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{n!} = \sum_{n=0}^{\infty} \frac{(3x)^n}{n!}$

RATIO TEST...

$$\lim_{n\to\infty} \left| \frac{(3x)^{n+1}}{(n+1)!} \cdot \frac{n!}{(3x)^n} \right| = \lim_{n\to\infty} \frac{|3x|}{n+1} = |3x| \lim_{n\to\infty} \frac{1}{n+1}$$

$$= |3x| \cdot 0 = 0$$

ABS. CONVERGENCE WHEN $0 < 1$

I.E., ALWAYS!