Math 132 - Quiz 8

May 7, 2020

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary. You must work individually on this quiz. This quiz is due no later than May 12.

1. (2 points) Eliminate the parameter θ to obtain an equation in x and y. Describe the graph of the resulting equation.

$$x = -5\sin\theta, \qquad y = -\cos\theta$$

$x^2 = 25\sin^2\theta \qquad y^2 = \cos^2\theta$

$\frac{x^2}{25} + y^2 = \sin^2\theta + \cos^2\theta = 1$

$\boxed{\frac{x^2}{25} + y^2 = 1}$

ELLIPSE WITH CENTER AT (0,0) AND VERTICES (±5,0) & (0,±1).

2. (2 points) Find a set of parametric equations for the line segment described by

$$y = 3x + 2, \quad 1 \le x \le 4.$$

$x = t$
$y = 3t + 2$ $\quad 1 \le t \le 4$

OR MAYBE

$x = \frac{t-2}{3}$ $\quad 5 \le t \le 14$
$y = t$

3. (2 points) Consider the curve described by the parametric equations

$$x = t^2, \quad y = \frac{1}{5}t^3, \qquad 0 \le t \le 2.$$

Find the area of the region between curve and the x-axis.

$$\text{AREA} = \int_0^2 \left(\frac{1}{5}t^3\right)(2t)\,dt = \int_0^2 \frac{2}{5}t^4\,dt$$

$$= \frac{2}{25}t^5 \Big|_0^2 = \frac{64}{25}$$

4. (2 points) Convert the point $(x, y) = (-5, -2)$ to polar coordinates.

↑ 3RD QUAD.

$$r^2 = (-5)^2 + (-2)^2 = 29$$

$\tan\theta = \frac{2}{5}$. SINCE $\tan^{-1}\left(\frac{2}{5}\right)$ IS IN 1ST QUAD,

LET'S USE $\left(-\sqrt{29}, \tan^{-1}\left(\frac{2}{5}\right)\right)$ OR $\left(\sqrt{29}, \tan^{-1}\left(\frac{2}{5}\right)+\pi\right)$

5. (2 points) Convert the polar equation $r = -3\cos\theta$ to an equation in rectangular coordinates.

$$r^2 = -3r\cos\theta$$

$$x^2 + y^2 = -3x$$

$$x^2 + 3x + y^2 = 0$$

$$x^2 + 3x + \frac{9}{4} + y^2 = \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 + y^2 = \left(\frac{3}{2}\right)^2$$