Math 132 - Test 2
March 12, 2020

Name key

Score ______

Show all work to receive full credit. Supply explanations where necessary. Evaluate all integrals by hand.

1. (10 points) Integrate: $\displaystyle\int_0^{\pi/2} 7\sin^3\theta\,\cos^4\theta\,d\theta$

$$7\int_0^{\pi/2} \sin^2\theta\cos^4\theta\sin\theta\,d\theta = 7\int_0^{\pi/2}(1-\cos^2\theta)\cos^4\theta\sin\theta\,d\theta$$

LET $u = \cos\theta$ $\qquad \theta = 0 \Rightarrow u = 1$

$du = -\sin\theta\,d\theta$ $\qquad \theta = \frac{\pi}{2} \Rightarrow u = 0$

$-du = \sin\theta\,d\theta$

$$-7\int_1^0 (1-u^2)u^4\,du$$

$$= 7\int_0^1 (1-u^2)u^4\,du = 7\int_0^1 (u^4-u^6)\,du = \left(\frac{7}{5}u^5 - u^7\right)\Big|_0^1$$

$$= \left(\frac{7}{5}-1\right) - 0 = \boxed{\frac{2}{5}}$$

2. (8 points) Integrate: $\displaystyle\int \cos^{-1}x\,dx$.

INT. BY PARTS...

$u = \cos^{-1}x \qquad du = \dfrac{-1}{\sqrt{1-x^2}}\,dx$

$dv = dx \qquad v = x$

$$x\cos^{-1}x + \int \frac{x}{\sqrt{1-x^2}}\,dx = x\cos^{-1}x - \frac{1}{2}\int u^{-1/2}\,du$$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$$= x\cos^{-1}x - \sqrt{u} + C$$

$$= \boxed{x\cos^{-1}x - \sqrt{1-x^2} + C}$$

3. (10 points) Integrate: $\int 5x^3 e^{2x}\, dx$

INT. BY PARTS. TABULAR METHOD IS BEST...

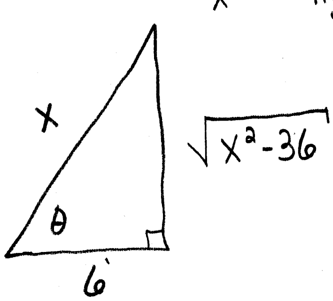
SIGNS	DERIVATIVES	ANTI-DERIVATIVES
$+$	$5x^3$	e^{2x}
$-$	$15x^2$	$\frac{1}{2}e^{2x}$
$+$	$30x$	$\frac{1}{4}e^{2x}$
$-$	30	$\frac{1}{8}e^{2x}$
$+$	0	$\frac{1}{16}e^{2x}$

$$\frac{5}{2}x^3e^{2x} - \frac{15}{4}x^2e^{2x} + \frac{15}{4}xe^{2x} - \frac{15}{8}e^{2x} + C$$

4. (6 points) After making the trigonometric substitution $x = 6\sec\theta$, you evaluated an integral and obtained $\theta + \cot\theta + C$. Resubstitute and write your result in terms of the variable x.

$$\frac{x}{6} = \sec\theta \Rightarrow \frac{6}{x} = \cos\theta$$

$$\frac{6}{x} = \frac{\text{ADJ}}{\text{HYP}}$$



$$\theta + \cot\theta + C$$

$$= \cos^{-1}\left(\frac{6}{x}\right) + \frac{6}{\sqrt{x^2-36}} + C$$

5. (6 points) Use a product-to-sum formula to evaluate the following integral.

$$\int \cos(3x)\cos(7x)\,dx$$

$$\cos(3x)\cos(7x) = \frac{1}{2}\left[\cos(3x-7x) + \cos(3x+7x)\right]$$

$$= \frac{1}{2}\left[\cos(-4x) + \cos(10x)\right]$$

$$= \frac{1}{2}\cos 4x + \frac{1}{2}\cos 10x$$

$$\frac{1}{2}\int \cos 4x\,dx + \frac{1}{2}\int \cos 10x\,dx$$

$$= \frac{1}{8}\sin 4x + \frac{1}{20}\sin 10x + C$$

6. (8 points) Integrate: $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$

SIMPLE SUBS...

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int u^2\,du = \frac{1}{3}u^3 + C$$

$$= \frac{1}{3}(\ln x)^3 + C$$

7. (8 points) Integrate: $\int \sec^6 8y \tan 8y \, dy$

$$\int \sec^5 8y \sec 8y \tan 8y \, dy$$

$u = \sec 8y$

$du = 8 \sec 8y \tan 8y \, dy$

$$\frac{1}{8} \int u^5 \, du = \frac{1}{48} u^6 + C = \boxed{\frac{1}{48} \sec^6 8y + C}$$

8. (8 points) In the following integral, carry out the appropriate trigonometric substitution, simplify the integrand, and then stop. Do not evaluate the new integral.

$$\int \frac{\sqrt{4-25x^2}}{x} \, dx$$

$a = 2$

$u = 5x$

$5x = 2 \sin \theta \Rightarrow x = \frac{2}{5} \sin \theta$ $\quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

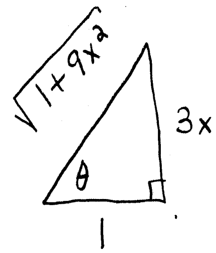
$dx = \frac{2}{5} \cos \theta \, d\theta$

$$\int \frac{\sqrt{4 - 4\sin^2\theta}}{\frac{2}{5} \sin \theta} \left(\frac{2}{5} \cos \theta\right) d\theta = \int \frac{\sqrt{4\cos^2\theta}}{\sin \theta} \cos \theta \, d\theta$$

$$= \int \frac{|2\cos\theta|}{\sin\theta} \cos\theta \, d\theta = \boxed{\int \frac{2\cos^2\theta}{\sin\theta} \, d\theta}$$

$|\cos\theta| = \cos\theta$

9. (10 points) Integrate: $\displaystyle\int \frac{dx}{\sqrt{1+9x^2}}$



TRIG. SUB... LET $3x = \tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$dx = \frac{1}{3}\sec^2\theta \, d\theta$$

$$\int \frac{\frac{1}{3}\sec^2\theta \, d\theta}{\sqrt{1+\tan^2\theta}} = \frac{1}{3}\int \frac{\sec^2\theta}{\sqrt{\sec^2\theta}} d\theta = \frac{1}{3}\int \frac{\sec^2\theta}{|\sec\theta|} d\theta$$

FOR $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, $\sec\theta > 0$ SO THAT $|\sec\theta| = \sec\theta$.

$$\frac{1}{3}\int \sec\theta \, d\theta = \frac{1}{3}\ln|\sec\theta + \tan\theta| + C$$

$$= \boxed{\frac{1}{3}\ln\left|\sqrt{1+9x^2} + 3x\right| + C}$$

10. (6 points) Integrate: $\displaystyle\int \cos^3 x \, dx$

$$\int \cos^2 x \cos x \, dx = \int (1-\sin^2 x)\cos x \, dx$$

$$u = \sin x$$
$$du = \cos x \, dx$$

$$\int (1-u^2)\,du = u - \frac{u^3}{3} + C$$

$$= \boxed{\sin x - \frac{1}{3}\sin^3 x + C}$$

OR YOU COULD USE THE POWER REDUCING FORMULA $\cos^3 x = \frac{3}{4}\cos x + \frac{1}{4}\cos 3x$.

Intentionally blank.

$\sin 2\theta = 2 \sin\theta \cos\theta$

You must work individually on the following problems. They are due by 7:35am on Tuesday, March 24.

11. (7 points) Integrate: $\int \sqrt{25 - 4x^2}\, dx$.

Trig. sub. $a = 5$, $u = 2x$

$$2x = 5\sin\theta \,, \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

or

$$x = \frac{5}{2}\sin\theta$$

$$dx = \frac{5}{2}\cos\theta \, d\theta$$

$$\int \sqrt{25 - 25\sin^2\theta}\left(\frac{5}{2}\right)\cos\theta \, d\theta$$

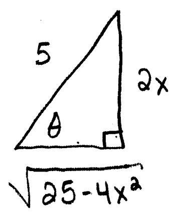
$$= \int \underbrace{\sqrt{25\cos^2\theta}}_{|5\cos\theta| = 5\cos\theta}\left(\frac{5}{2}\right)\cos\theta \, d\theta$$

$$\frac{25}{2}\int \cos^2\theta \, d\theta$$

$$= \frac{25}{4}\int (1 + \cos 2\theta)\, d\theta$$

$$= \frac{25}{4}\theta + \frac{25}{8}\sin 2\theta + C$$

$$= \frac{25}{4}\theta + \frac{25}{4}\sin\theta\cos\theta + C$$



$$\frac{25}{4}\left[\sin^{-1}\left(\frac{2x}{5}\right) + \frac{2x}{5}\,\frac{\sqrt{25-4x^2}}{5}\right] + C$$

12. (3 points) Write the form of the partial fraction decomposition of $\frac{x}{x^3(x^2+9)^2(2x+1)}$.

Do not solve for the undetermined coefficients.

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+9} + \frac{Fx+G}{(x^2+9)^2} + \frac{H}{2x+1}$$

13. (10 points) Integrate: $\displaystyle\int \frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8}\,dx$

(Use a PFD.)

$$\begin{array}{r} 2x \\ x^2 - 2x - 8 \,\big)\, 2x^3 - 4x^2 - 15x + 5 \\ -\left(2x^3 - 4x^2 - 16x\right) \\ \hline x + 5 \end{array}$$

So, the integrand is

$$2x + \frac{x+5}{(x-4)(x+2)}$$

↙

PFD

$$\frac{x+5}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$$

$$x + 5 = A(x+2) + B(x-4)$$

$x = -2$: $3 = -6B \Rightarrow B = -\frac{1}{2}$

$x = 4$: $9 = 6A \Rightarrow A = \frac{3}{2}$

$$\int \left(2x + \frac{3/2}{x-4} - \frac{1/2}{x+2}\right) dx$$

$$= \boxed{x^2 + \frac{3}{2}\ln|x-4| - \frac{1}{2}\ln|x+2| + C}$$