Math 132 - Final Exam
May 12, 2020

Name key

Score ____________

Show all work to receive full credit. This test is due no later than May 16 at 8 am.

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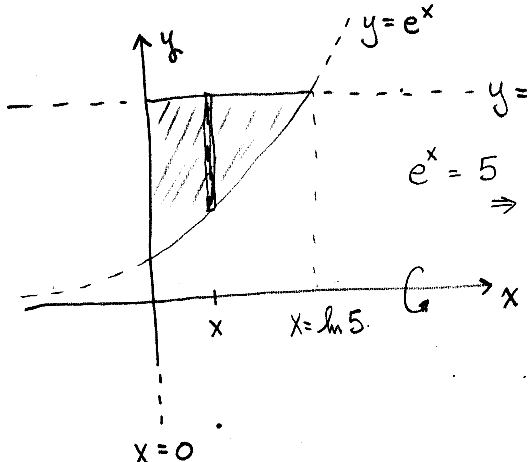
1. The region bounded by the graphs of $y = e^x$, $y = 5$, and $x = 0$ is rotated about the x-axis to form a solid. Set up the definite integral that gives the volume of the solid. Then use your calculator (or computer) to evaluate the integral.



$e^x = 5 \Rightarrow x = \ln 5$

WASHERS...

$$V = \int_0^{\ln 5} \pi(25 - e^{2x})\,dx$$

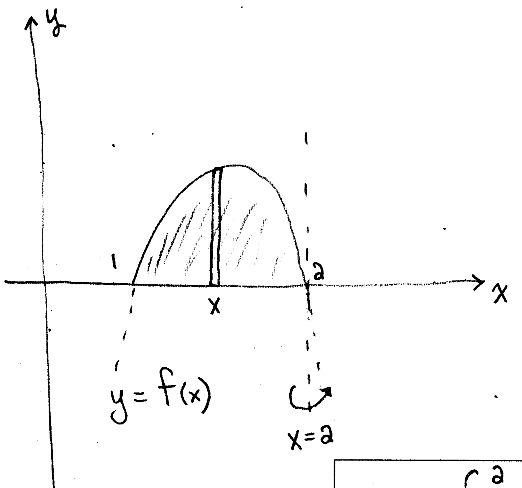
$$\boxed{\int_0^{\ln 5} \pi(25 - e^{2x})\,dx = \pi(25\ln 5 - 12)}$$

DECIMAL...

88.70585

2. The bounded region between the graph of $f(x) = -x^2+3x-2$ and the x-axis is rotated about the line $x = 2$ to form a solid. Set up the definite integral that gives the volume of the solid. Then use your calculator (or computer) to evaluate the integral.

$f(x) = -x^2+3x-2 = -(x-2)(x-1)$



SHELLS...

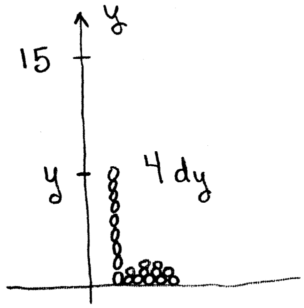
$$V = 2\pi\int_1^2 (2-x)(-x^2+3x-2)\,dx$$

$$\boxed{2\pi\int_1^2 (2-x)(-x^2+3x-2)\,dx = \frac{\pi}{6}}$$

DECIMAL...

0.52360

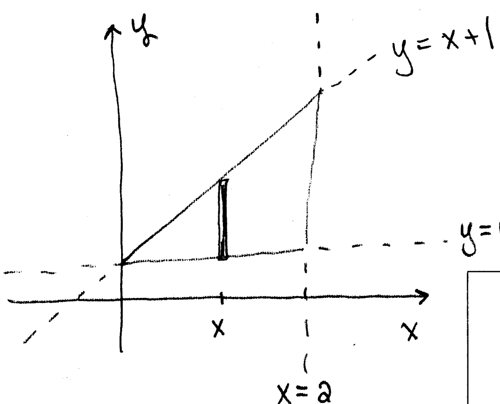
3. A 15-foot chain weighing 4 pounds per foot is lying on the ground. How much work is required to raise one end of the chain to a height of 15 feet so that it is hanging fully extended?



$$W = \int_0^{15} 4y\,dy = 2y^2\Big|_0^{15} = 2(15)^2 = 450 \text{ FT-LB}$$

$$\boxed{450 \text{ FT-LB}}$$

4. A thin plate lies in the 1st quadrant bounded by the graphs of $y = x + 1$, $y = 1$, and $x = 2$. The density of the plate at the point (x, y) is given by $\rho(x) = 2 + \sqrt{x}$. Set up the definite integral that gives the moment about the x-axis. Do not evaluate.



$$dm = (x+1-1)(2+\sqrt{x})\,dx = x(2+\sqrt{x})\,dx$$

$$M_x = \int_0^2 y\,dm = \int_0^2 \left(\frac{x+1+1}{2}\right) dm$$

$$\boxed{M_x = \int_0^2 \left(\frac{x+2}{2}\right)(x)(2+\sqrt{x})\,dx}$$

5. Integrate: $\displaystyle\int x^5 \ln x\,dx$

$u = \ln x \qquad du = \frac{1}{x}\,dx$

$dv = x^5\,dx \qquad v = \frac{1}{6}x^6$

$$\frac{1}{6}x^6 \ln x - \int \frac{1}{6}x^5\,dx$$

$$\boxed{\frac{1}{6}x^6 \ln x - \frac{1}{36}x^6 + C}$$

6. Given the definite integral $\int_0^3 \frac{x^3}{\sqrt{x^2+9}}\,dx$, carry out the appropriate trigonometric substitution (including the integration bounds), simplify the new integrand, and stop. Do not evaluate.

$x = 3\tan\theta \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$dx = 3\sec^2\theta\, d\theta$

$x = 0 \Rightarrow \tan\theta = 0$
$\Rightarrow \theta = 0$

$x = 3 \Rightarrow \tan\theta = 1$
$\Rightarrow \theta = \frac{\pi}{4}$

$$\int_0^{\pi/4} \frac{27\tan^3\theta}{\underbrace{\sqrt{9\tan^2\theta+9}}_{3|\sec\theta| = 3\sec\theta}}\, 3\sec^2\theta\, d\theta$$

$$\boxed{\int_0^{\pi/4} 27\tan^3\theta\,\sec\theta\, d\theta}$$

7. Determine the partial fraction decomposition of $\frac{12-5x-2x^2}{(x+1)(x^2+4)}$.

$$\frac{12-5x-2x^2}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4}$$

$12-5x-2x^2 = A(x^2+4) + (Bx+C)(x+1)$

$x=-1 \Rightarrow 15 = 5A$

$x = 0 \Rightarrow 12 = 4A + C$

$x = 1 \Rightarrow 5 = 5A + 2(B+C)$

$A = 3$

$C = 0$

$B = -5$

$$\boxed{\frac{3}{x+1} - \frac{5x}{x^2+4}}$$

8. Use the trapezoid rule with five subintervals ($n = 5$) to approximate $\int_0^1 e^{x^2}\,dx$.

$\Delta x = \frac{1}{5}$

$x_0 = 0,\ x_1 = 0.2,\ x_2 = 0.4,\ x_3 = 0.6,\ x_4 = 0.8,\ x_5 = 1$

$$T = \frac{\Delta x}{2}\cdot\left(e^0 + 2e^{0.04} + 2e^{0.16} + 2e^{0.36} + 2e^{0.64} + e^1\right)$$

≈ 1.48065

$$\boxed{1.48065}$$

9. Explain why the series $\sum_{n=0}^{\infty} 1$ diverges. Do not apply a test, but rather use what it means for a series to converge or diverge.

PARTIAL SUMS... $S_0 = 1$, $S_1 = 1 + 1 = 2$, $S_2 = 3$, $S_3 = 4$, $\ldots$ $S_n = n+1$

$$\boxed{S_n = n+1, \quad \lim_{n\to\infty} S_n = \infty \Rightarrow \text{SERIES DIVERGES}}$$

10. Carefully read the integral test in the class notes. In order to apply the integral test to the series $\sum_{n=0}^{\infty} \frac{1}{n^2 - 3n + 4}$, what improper integral must be evaluated? Do not evaluate.

LET $f(x) = \dfrac{1}{x^2 - 3x + 4}$. LOOK AT THE GRAPH OF f. WE SEE THAT f IS POSITIVE, CONTINUOUS, AND DECREASING FOR $x \geq 2$

$$\boxed{\int_2^{\infty} \frac{1}{x^2 - 3x + 4}\, dx}$$

11. You plan to use the limit comparison test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n^2 - n + 4}{2n^3 + 17n}$. What is a good series to use for your comparison, and will it show convergence or divergence?

$\dfrac{n^2 - n + 4}{2n^3 + 17n}$ "LOOKS LIKE" $\dfrac{n^2}{2n^3}$ FOR LARGE n

$$\frac{n^2}{2n^3} = \frac{1}{2n}$$

$$\boxed{\text{COMPARE WITH } \sum_{n=1}^{\infty} \frac{1}{n}. \quad \text{SERIES DIVERGES}}$$

12. Give an example of a series that converges conditionally, and name the tests you would use to prove it.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \quad \text{CONVERGES BY AST} \qquad \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \quad \text{DIVERGES } p=\tfrac{1}{2} \text{ SERIES}$$

$$\boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}, \quad \text{ALTERNATING SERIES TEST \& P-SERIES TEST}}$$

13. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(2x-1)^n}{4^n n}$.

RATIO TEST ...

$$\lim_{n\to\infty} \left| \frac{(2x-1)^{n+1}}{4^{n+1}(n+1)} \cdot \frac{4^n n}{(2x-1)^n} \right| = |2x-1| \lim_{n\to\infty} \frac{n}{4(n+1)} = \frac{|2x-1|}{4} < 1$$

$$|2x-1| < 4$$

$$|x-\tfrac{1}{2}| < 2$$

$$\boxed{R = 2}$$

14. Find a power series for $f(x) = \frac{1}{1+x} - \frac{1}{1-x}$.

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n \qquad f(x) = \sum_{n=0}^{\infty} (-x)^n - x^n$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad = -2x - 2x^3 - 2x^5 - \cdots$$

$$\boxed{\sum_{n=0}^{\infty} -2x^{2n+1}}$$

15. Determine the 3rd Maclaurin polynomial for $g(x) = \sqrt{x+1}$. Then use it to approximate $g(0.25)$.

$g(x) = (x+1)^{1/2} \quad g(0) = 1$

$g'(x) = \frac{1}{2}(x+1)^{-1/2} \quad g'(0) = \frac{1}{2}$

$g''(x) = -\frac{1}{4}(x+1)^{-3/2} \quad g''(0) = -\frac{1}{4}$

$g'''(x) = \frac{3}{8}(x+1)^{-5/2}$

$g'''(0) = \frac{3}{8}$

$P_3(x) = 1 + \frac{1}{2}x - \frac{1/4}{2}x^2 + \frac{3/8}{3!}x^3$

$$\boxed{\begin{array}{l} P_3(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 \\ g(0.25) \approx P_3(0.25) = 1.11816 \end{array}}$$

16. Determine the Maclaurin series for $f(x) = e^{x/2}$.

$f(x) = e^{x/2}, \quad f'(x) = \frac{1}{2}e^{x/2}, \quad \dots, \quad f^{(n)}(x) = \frac{1}{2^n}e^{x/2}$

$f^{(n)}(0) = \frac{1}{2^n} \Rightarrow \sum_{n=0}^{\infty} \frac{1}{2^n n!}x^n$

$$\boxed{\sum_{n=0}^{\infty} \frac{x^n}{2^n n!}}$$

17. Eliminate the parameter θ to obtain an equation in x and y. Describe the graph of the resulting equation.

$$x = 2 - 3\sin\theta, \qquad y = -5 + 3\cos\theta$$

$(x-2)^2 = 9\sin^2\theta \qquad (y+5)^2 = 9\cos^2\theta$

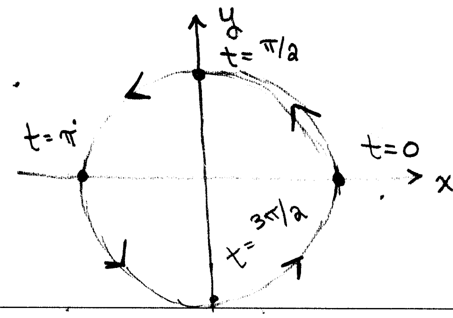
$(x-2)^2 + (y+5)^2 = 9$

$$\boxed{(x-2)^2 + (y+5)^2 = 9 \quad \text{CIRCLE OR RADIUS 3 CENTERED AT } (2,-5)}$$

18. The parametric equations $x = \cos t$, $y = \sin t$ describe the unit circle. If we use the area formula for parametric curves to compute the area of the unit circle, we get

$$\int_0^{2\pi} (\sin t)(-\sin t)\,dt = -\pi.$$

Why is the answer negative?



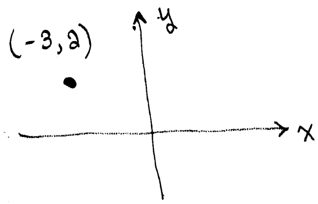
THE CURVE IS

COUNTER-CLOCKWISE

ORIENTED

$\boxed{\text{THE CURVE IS COUNTER-CLOCKWISE ORIENTED. THE FORMULA USED IS FOR CLOCKWISE CURVES.}}$

19. Convert the point $(x, y) = (-3, 2)$ to polar coordinates.



$\leftarrow$ 2ND QUAD.

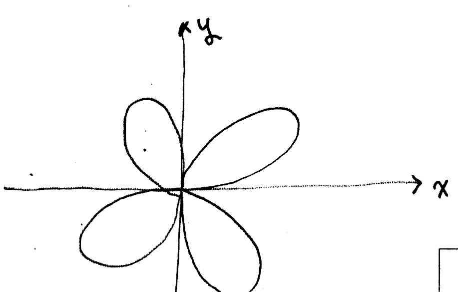
$r^2 = (-3)^2 + 2^2 = 13$

$\tan\theta = \frac{2}{-3}$ $\quad \tan^{-1}\left(-\frac{2}{3}\right)$ IS IN 4TH QUAD.

$$\boxed{\left(\sqrt{13},\ \tan^{-1}\left(-\tfrac{2}{3}\right) + \pi\right)}$$

DECIMAL...
$(3.60555,\ 2.55359)$

20. Set up the definite integral that gives the length of one petal of the rose curve defined by the polar equation $r = 4\sin(2\theta)$. Do not evaluate.



$\theta = 0 \Rightarrow r = 0$

$\theta = \frac{\pi}{2} \Rightarrow r = 0$

$r' = 8\cos 2\theta$

$$L = \int_0^{\pi/2} \sqrt{(r)^2 + (r')^2}\,d\theta$$

$$\boxed{\int_0^{\pi/2} \sqrt{16\sin^2 2\theta + 64\cos^2 2\theta}\,d\theta}$$