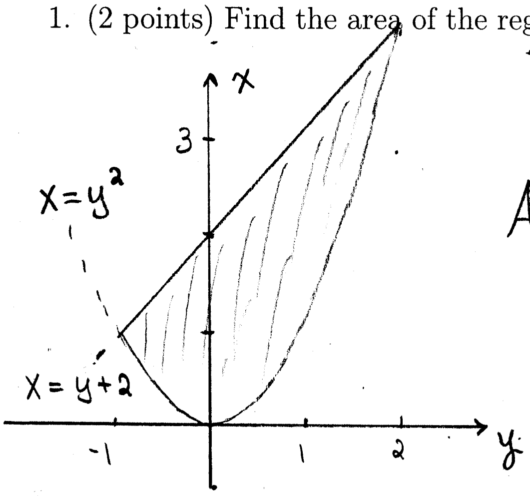
Math 132 - Homework 1
February 3, 2021

Name key
Score

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due on February 10.

1. (2 points) Find the area of the region bounded by the graphs of $y^2 = x$ and $x = y + 2$.



$$\text{Area} = \int_{-1}^{2} (y+2-y^2)\,dy$$

$$= \frac{1}{2}y^2 + 2y - \frac{1}{3}y^3 \Big|_{-1}^{2}$$

$$= \left(2+4-\frac{8}{3}\right) - \left(\frac{1}{2}-2+\frac{1}{3}\right)$$

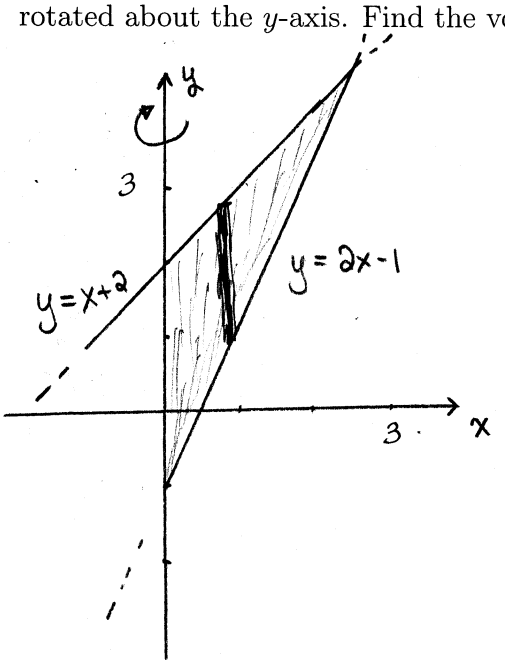
$$= \boxed{\frac{9}{2}}$$

$y^2 = y+2$

$y^2 - y - 2 = 0 \quad (y-2)(y+1) = 0$

$y=2, \; y=-1$

2. (2 points) The region bounded by the graphs of $y = x + 2$, $y = 2x - 1$, and $x = 0$ is rotated about the y-axis. Find the volume of the solid that is generated.



Shells...

$$\text{Volume} = 2\pi \int_0^3 x\left((x+2)-(2x-1)\right)dx$$

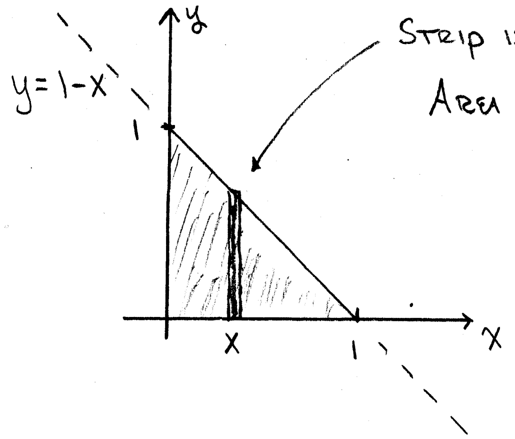
$$= 2\pi \int_0^3 (3x - x^2)\,dx = 2\pi\left(\frac{3}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^3$$

$$= 2\pi\left(\frac{27}{2} - \frac{27}{3}\right) = \boxed{9\pi}$$

$x+2 = 2x-1$

$3 = x$

3. (2 points) The base of a solid is the triangle with vertices $(0,0)$, $(1,0)$, and $(0,1)$. Slices perpendicular to the x-axis are semicircles. Find the volume of the solid.



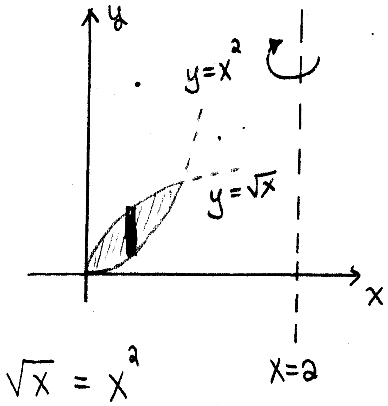
STRIP IS BASE OF A SEMICIRCLE.

$$\text{AREA AT } x = \frac{1}{2}\pi(\text{RADIUS})^2 = \frac{1}{2}\pi\left(\frac{1-x}{2}\right)^2 = \frac{1-2x+x^2}{8}\pi$$

$$\text{VOLUME} = \int_0^1 \frac{\pi}{8}\left(1-2x+x^2\right)dx$$

$$= \frac{\pi}{8}\left(x - x^2 + \frac{1}{3}x^3\right)\Big|_0^1 = \boxed{\frac{\pi}{24}}$$

4. (2 points) The region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ is rotated about the line $x = 2$. Find the volume of the solid that is generated.



SHELLS...

$$\text{VOLUME} = 2\pi\int_0^1 (2-x)\left(\sqrt{x}-x^2\right)$$

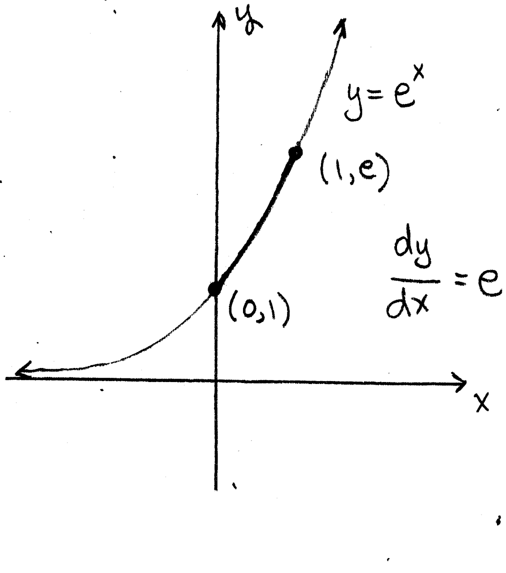
$$= 2\pi\int_0^1 \left(2x^{1/2} - 2x^2 - x^{3/2} + x^3\right)dx$$

$\sqrt{x} = x^2$

$\Rightarrow x = 0, 1$

$$= 2\pi\left(\frac{4}{3}x^{3/2} - \frac{2}{3}x^3 - \frac{2}{5}x^{5/2} + \frac{1}{4}x^4\right)\Big|_0^1 = 2\pi\left(\frac{2}{3} - \frac{2}{5} + \frac{1}{4}\right) = \boxed{\frac{31\pi}{30}}$$

5. (2 points) Find the length of the graph of $y = e^x$ over the interval from $x = 0$ to $x = 1$. Use technology to evaluate the definite integral.



$$\frac{dy}{dx} = e^x$$

$$\text{ARC LENGTH} = \int_0^1 \sqrt{1+(e^x)^2}\,dx$$

$$\boxed{= \int_0^1 \sqrt{1+e^{2x}}\,dx \approx 2.0035}$$