# Math 132 - Homework 2

February 17, 2021

Name __key__

Score _____

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due on February 24.

1. (2 points) A rectangular dam is 40 feet high and 60 feet wide. Using $62.4\,\text{lb/ft}^3$ as the weight density of the water, compute the total force on the dam when

   (a) the surface of the water is at the top of the dam.
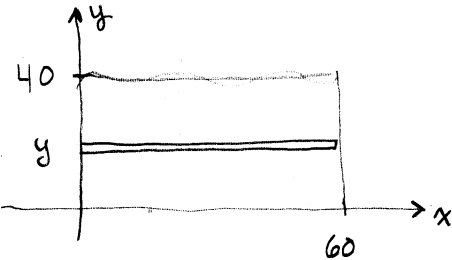
$$62.4 \int_0^{40} (40-y)(60)\,dy$$

$$= 62.4 \left(2400y - 30y^2 \right) \Big|_0^{40}$$

$$= 62.4 (48000) = \boxed{2,995,200 \text{ lb}}$$

   (b) the surface of the water is halfway down the dam.

$$62.4 \int_0^{20} (20-y)(60)\,dy$$

$$= 62.4 \left(1200y - 30y^2 \right) \Big|_0^{20}$$

$$= 62.4 (12000) = \boxed{748,800 \text{ lb}}$$

2. (2 points) Use the definition of $\cosh(x)$ in terms of exponential functions to prove that $\int \cosh(x)\,dx = \sinh(x) + C$.

$$\int \cosh x \, dx = \frac{1}{2} \int (e^x + e^{-x})\,dx = \frac{1}{2}(e^x - e^{-x}) + C$$

$$= \sinh x + C \quad \checkmark$$

$$\int e^{-x}\,dx = -\int e^u\,du$$

$$u = -x$$
$$-du = dx \qquad = -e^{-x} + C$$

Turn over.

3. (2 points) Integrate: $\int x^2 e^{-x^3} dx = -\frac{1}{3} \int e^u \, du$

$u = -x^3$

$du = -3x^2 \, dx$

$-\frac{1}{3} du = x^2 dx$

$= -\frac{1}{3} e^u + C$

$= \boxed{-\frac{1}{3} e^{-x^3} + C}$

4. (2 points) Integrate: $\int \tan^{-1} x \, dx = X \text{TAN}^{-1} X - \int \frac{X}{1+X^2} dx$

$u = \text{TAN}^{-1} X \qquad du = \frac{1}{1+X^2} dx$

$dv = dx \qquad V = X$

$u = 1 + x^2$

$du = 2x \, dx$

$\frac{1}{2} du = x \, dx$

$\int \frac{X}{1+X^2} dx = \frac{1}{2} \int \frac{1}{u} du$

$= \frac{1}{2} \ln |u| + C$

$\boxed{\text{ANS:} \quad X \text{TAN}^{-1} X - \frac{1}{2} \ln |1+x^2| + C}$

5. (2 points) Integrate: $\int x^2 \cos x \, dx$

SIGNS	U & DERIVS	$\frac{dv}{dx}$ & ANTIS
+	x^2	COS X
−	$2x$	SIN X
+	2	− COS X
−	0	− SIN X

$= \boxed{X^2 \text{SIN} X + 2X \cos X - 2 \text{SIN} X + C}$