Math 132 - Homework 4

March 31, 2021

Name key

Score

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due April 7.

1. (2 points) Use the trapezoidal rule with $n = 6$ to approximate $\int_0^3 \frac{1}{1+x^3}\,dx$.

$h = \frac{3-0}{6} = \frac{1}{2}$ $\qquad f(x) = \frac{1}{1+x^3}$

$x_0 = 0$

$x_1 = 0.5$

$x_2 = 1$

$x_3 = 1.5$

$x_4 = 2$

$x_5 = 2.5$

$x_6 = 3$

$$\int_0^3 \frac{1}{1+x^3}\,dx \approx \frac{1/2}{2}\Big[f(0) + 2f(0.5) + 2f(1) + 2f(1.5) + 2f(2) + 2f(2.5) + f(3)\Big]$$

$$= \boxed{1.15328947}$$

2. (2 points) Rewrite as a limit and evaluate: $\int_1^\infty xe^{-x}\,dx$

$$\lim_{t\to\infty} \int_1^t xe^{-x}\,dx = \lim_{t\to\infty}\Big[xe^{-x} + e^{-x}\Big]_t^1$$

$$= \lim_{t\to\infty}\left[\frac{2}{e} - te^{-t} - e^{-t}\right]$$

$$= \frac{2}{e} - \lim_{t\to\infty}\frac{t}{e^t} - \lim_{t\to\infty}\frac{1}{e^t}$$

$$= \frac{2}{e} - 0 - 0 = \boxed{\frac{2}{e}}$$

$\int xe^{-x}\,dx = -xe^{-x} - e^{x}$

+	x	e^{-x}
−	1	$-e^{-x}$
+	0	e^{-x}

L'HôPITAL'S RULE

$\lim_{t\to\infty}\frac{t}{e^t} = \lim_{t\to\infty}\frac{1}{e^t}$

$= 0$

Turn over.

3. (2 points) Rewrite as a limit and evaluate: $\int_0^1 \frac{1}{1-x}\,dx$

$$\lim_{t\to 1^-} \int_0^t \frac{1}{1-x}\,dx = \lim_{t\to 1^-} -\ln|1-x| \Big|_0^t$$

$$= \lim_{t\to 1^-}\left(-\ln(1-t) + \cancel{\ln 1}^{\,0}\right) = \lim_{t\to 1^-} -\ln(1-t)$$

$$= -(-\infty)$$

$$= \boxed{+\infty}$$

4. (1 point) Find an explicit formula for a_n where $a_1 = 1$ and $a_n = a_{n-1} + n$ for $n \geq 2$.

$a_1 = 1$

$a_2 = 1+2 = 3$

$a_3 = 3+3 = 6$

$a_4 = 6+4 = 10$

$a_5 = 10+5 = 15$

$$\boxed{a_n = \frac{n(n+1)}{2}}$$

THESE ARE CALLED TRIANGULAR NUMBERS.

5. (1 point) Find the limit of the sequence with $a_n = \frac{\sqrt{n}}{\sqrt{n+1}}$.

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+1}} = \sqrt{\lim_{n\to\infty} \frac{n}{n+1}} = \sqrt{\lim_{n\to\infty} \frac{1}{1+\frac{1}{n}}} = \sqrt{1}$$

$$= \boxed{1}$$

6. (2 points) Evaluate the series or show that it diverges: $\sum_{n=1}^{\infty} [2^{1/n} - 2^{1/(n+1)}]$

THIS IS A TELESCOPING SERIES: $(2 - 2^{1/2}) + (2^{1/2} - 2^{1/3}) + (2^{1/3} - 2^{1/4}) + \cdots$

$S_1 = 2 - 2^{1/2}$

$S_2 = 2 - 2^{1/3}$

$S_3 = 2 - 2^{1/4}$

$S_4 = 2 - 2^{1/5}$

IN GENERAL, $S_n = 2 - 2^{1/n+1}$

$$\lim_{n\to\infty}\left(2 - 2^{1/n+1}\right) = 2 - 2^0 = \boxed{1}$$

$$\boxed{\sum_{n=1}^{\infty}\left(2^{1/n} - 2^{1/n+1}\right) = 1}$$