Math 132 - Homework 5

April 28, 2021

Name key

Score

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due May 5.

1. (2 points) Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$. Use the alternating series remainder theorem to find a value of N that guarantees that the sum of the first N terms differs from the sum of the series by no more than 10^{-6}.

WE NEED $\frac{1}{(n+1)^2} < 10^{-6}$ OR $(n+1)^2 > 10^6$

OR $n+1 > 10^3$

$\boxed{\text{USE } N = 1000.}$ THEN THE ERROR IS LESS THAN $\frac{1}{(1001)^2}$.

2. (2 points) Find the radius and interval of convergence: $\sum_{n=1}^{\infty} \frac{(2x)^n}{n}$

RATIO TEST ...

$$\lim_{n\to\infty} \left| \frac{(2x)^{n+1}}{n+1} \cdot \frac{n}{(2x)^n} \right| = 2|x| \lim_{n\to\infty} \frac{n}{n+1} = 2|x| < 1$$

$$\Rightarrow |x| < \frac{1}{2}$$

RADIUS OF CONVERGENCE IS $\frac{1}{2}$.

SO FAR WE HAVE ABS. CONVERGENCE ON $\left(-\frac{1}{2}, \frac{1}{2}\right)$. CHECK ENDPTS.

$x = \frac{1}{2}$: $\sum_{n=1}^{\infty} \frac{1}{n}$ DIVERGES. (HARMONIC SERIES.)

$x = -\frac{1}{2}$: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ CONVERGES BY AST.

INTERVAL OF CONVERGENCE IS $\left[-\frac{1}{2}, \frac{1}{2}\right)$.

Turn over.

SERIES OF ABS. VALUES...

$\sum \frac{\sqrt{n+3}}{n}$ DIVERGES BY LIMIT COMPARISON WITH $p=\frac{1}{2}$ SERIES, $\sum \frac{1}{\sqrt{n}}$.

3. (6 points) Determine whether each series converges absolutely, conditionally, or not at all. Show your work.

(a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{n+3}}{n}$

$$a_n = \sqrt{\frac{n+3}{n^2}} = \sqrt{\frac{1}{n}+\frac{3}{n^2}}.$$

POSITIVE & EACH TERM GETS SMALLER AS n INCREASES. SO $0 < a_{n+1} < a_n$.

-AND- $\lim_{n\to\infty} a_n = 0$

SERIES CONVERGES BY AST.

CONDITIONAL CONVERGENCE.

(b) $\sum_{n=1}^{\infty}\frac{n^2}{2^n}$

RATIO TEST...

$$\lim_{n\to\infty}\left|\frac{(n+1)^2}{2^{n+1}}\cdot\frac{2^n}{n^2}\right| = \lim_{n\to\infty}\frac{1}{2}\frac{(n+1)^2}{n^2} = \frac{1}{2} < 1$$

SERIES CONVERGES ABSOLUTELY.

(c) $\sum_{k=1}^{\infty}\left(\frac{2k^2-1}{k^2+3}\right)^k$

ROOT TEST...

$$\lim_{k\to\infty}\sqrt[k]{\left|\frac{2k^2-1}{k^2+3}\right|^k} = \lim_{k\to\infty}\frac{2k^2-1}{k^2+3} = 2 > 1$$

SERIES DIVERGES.